● 不可使用電子計算機

Notation:

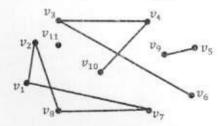|	
$\mathbb{Z}^+$	The set of **positive integers**
$\mathbb{N}$	The set of **natural numbers** (including 0)
$(F \circ G)(x)$	The **composition** of two functions $F(x)$ and $G(x)$
$\gcd(a, b)$	The **greatest common divisor** of two positive integers a and b
$S \cap T$	The **intersection** of two sets S and T
$S \cup T$	The **union** of two sets S and T
$\overline{S}$	The **complement** of a set S

一、填充題（每格5分，共70分）

1. Let $F: \mathbb{N} \to \mathbb{N}$, $F(x) = \int_0^x (9y^2 + 6y)dy$, and $G: \mathbb{N} \to \mathbb{N}$, $G(x) = x^2$.
   A. Is $F(x)$ a **one-to-one correspondence** function? _____ (yes or no)
   B. Is $(F \circ G)(x)$ an **onto** function? _____ (yes or no)

2. For a binary relation $R = \{(s_1, s_2) \mid s_1, s_2 \in \mathbb{Z}^+, \gcd(s_1, s_2) = 1\}$,
   A. Is R a **reflexive** binary relation? _____ (yes or no)
   B. Is R a **symmetric** binary relation? _____ (yes or no)

3. Let $S = \{x \in \mathbb{Z}^+ \mid x \le 8, \gcd(x, x + 10) = 1\}$ and $T = \{x \in \mathbb{Z}^+ \mid x \le 9, x \equiv 17 \pmod 4\}$.
   A. The **power set** of S is _____.
   B. The **Cartesian product** of S and T, i.e. $S \times T$, is _____.

4. For a **complete undirected** graph G with total n **vertices** ($n \in \mathbb{Z}^+$, $n \ge 2$),
   [Note: Use the common notation of permutations/combinations, and do not need to compute the real values.]
   A. The number of **edges** in G is _____.
   B. The number of different **sub-graphs**, with **exactly** n vertices, in G is _____.
   C. For $n = 4$, the total number of different **sub-graphs** in G is _____.

5. For a **complete** n-**ary** tree T of **height** h ($n, h \in \mathbb{Z}^+$, $n \ge 2$),
   A. The number of **vertices** in T is _____.
   B. The number of **edges** in T is _____.

6. Consider the following **undirected** graph G:

The **connected components** in G are _____.

7. Consider a set of poker cards (total **52** cards, with **4** suits {spades, hearts, diamonds, clubs} and **13** cards in each suit {2, 3, ..., 10, jack, queen, king, ace}). One person sequentially draws **13 cards** from this set of cards, i.e. he draws the 13 cards one by one, and never puts the drawn card back. If each card is **equally likely** to be drawn,

   [Note: Use the common notation of permutations/combinations, and do not need to compute the real values.]

   A. The probability that the drawn cards contain **exactly one** queen is _____.

   B. The probability that the drawn cards contain **at least three** kings is _____.

二、問答題（每題 15 分，共 30 分）

1. Let P, Q, and R denote different sets. Prove that $\overline{(P \cap Q) \cup (Q \cap R)} \cap \overline{Q \cup (P \cap R)}$ and $\overline{Q} \cap (\overline{P} \cup \overline{R})$ are equivalent using **set equivalence laws**.

   [Hint: For two sets S and T, $S \cup (S \cap T) = S$ and $S \cap (S \cup T) = S$ (absorption laws).]

2. Consider a **2D color** image of **resolution** $m \times n$. Each pixel in the image contains **four channels** (red, green, blue, and alpha), while the value of each channel is an **unsigned 8-bit integer**. How many different results can this image represent? Please explain your answer in detail.

   [Note: Use the common notation of permutations/combinations, and do not need to compute the real values.]