元智大學 101 學年度研究所 **碩士班** 招生試題卷

系(所)別：機械工程學系碩士班　組別：甲組　科目：工程數學　用紙第 1 頁共 2 頁

●不可使用電子計算機

1. Using Variation of Parameters to find a solution of the following equation. (16 %)

$$y'' - 4y' + 4y = (x+1)e^{2x}$$

2. Using the method of Laplace Transformation to solve the initial value problem of $y(t)$. (17 %)

$$y'' + 2y' + y = te^{-t} \quad \text{with} \quad y(0) = 1 \ , \ \left.\frac{dy}{dt}\right|_{t=0} = -2$$

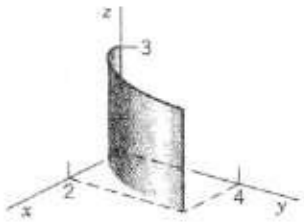
3. For the matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$,

(1) Find the inverse of **A** by Gauss-Jordan Method. (6 %)

(2) Verify $(A^2)^{-1} = (A^{-1})^2$ for **A**. (6 %)

(3) Prove the formula $(A^2)^{-1} = (A^{-1})^2$. (6 %)

4. Compute the flux of water through the parabolic cylinder $S: y = x^2, \ 0 \le x \le 2, \ 0 \le z \le 3$ (as below) if the velocity vector is $\vec{v} = \vec{F} = [3z^2, 6, 6xz]$, speed being measured in meters/sec. (Generally, $\vec{F} = \rho \vec{v}$, but water has the density ρ = 1 gm/cm^3.) (15 %)



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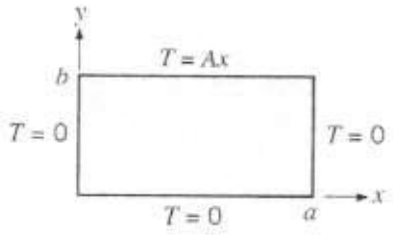
5. A two-dimensional rectangular plate is subjected to the boundary conditions shown as below. Derive an expression for the steady state temperature distributions T(x,y) with solving the heat conduction equation. (17 %)



The heat conduction equation is :

$$\frac{\partial^2 T(x,y)}{\partial x^2} + \frac{\partial^2 T(x,y)}{\partial y^2} = 0$$

Please find the solution in sin, cos, sinh, cosh series functions by the method of separation variables.

6. There is periodic square wave with analytic represented as $f(x)$ function

$$f(x) = \begin{cases} -k & \text{when} \quad -\pi < x < 0 \\ k & \text{when} \quad 0 < x < \pi \end{cases}$$

$$\text{and } f(x + 2\pi) = f(x)$$

Please find the Fourier coefficient of a_n, b_n and their series functions to present the $f(x)$ functions. (17 %)