元智大學 101 學年度研究所 **碩士班** 招生試題卷

系(所)別：生物與醫學資訊碩士學位學程　組別：不分組　科目：離散數學　用紙第 1 頁共 2 頁

●不可使用電子計算機

1 In the questions below consider all bit strings of length 12. (5%)

a) How many begin with 110?

b) How many begin with 11 or end with 10?

c) How many have exactly four 1s?

d) How many have exactly four 1s and none of these 1s are adjacent to each other?

2. Suppose |A| = 4 and |B| = 10, (5%)

a) Find the number of functions f: A→ B.

b) Find the number of 1-1 functions f: A→ B.

3. A game consisting of flipping a coin ends when the player gets two heads in a row, two tails in a row, or flips the coin four times,

a) Draw a tree diagram to show the ways in which the game can end.

b) In how many ways can the game end? (5%)

4. (5%)

a) Find the number of solutions to x + y + z = 32, where x,y,and z are nonnegative integers.

b) Answer part (a), but assume that x>=7 and y<=15.

5. (5%)

Assume that you have a bowl containing hard candies: 50 cherry, 50 strawberry, 40 orange, 70 lemon, and 40 pineapple. Assuming that the pieces of each flavor are identical.

a) How many handfuls of 15 are possible?

b) How many handfuls of 15 are possible with at least one piece of each flavor?

c) How many handfuls of 15 are possible with at least two piece of each flavor?

6. (5%)

a) What is the length of the longest simple circuit in W_{10}?

b) What is the length of the longest simple circuit in K_5?

c) What is the length of the longest simple circuit in $K_{4,10}$?

d) What is the length of the longest simple circuit in Q_3?

7. In the questions below either give an example or prove that there are none.

a) A simple graph with 6 vertices, whose degrees are 2,2,2,3,4,4.

b) A simple graph with 8 vertices, whose degrees are 0,1,2,3,4,5,6,7.

c) A simple graph with degrees 1,2,2,3.

d) A simple graph with degrees 2,3,4,4,4.

e) A graph with 7 vertices that has a Hamilton circuit but no Euler circuit.

f) A planar graph with 7 vertices, 9 edges, and 5 regions. (10%)

●不可使用電子計算機

8. (5%)

a) Give a recurrence relation for e_n = the number of edges of the graph K_n

b) Give a recurrence relation for v_n = the number of vertices of the graph Q_n

c) Give a recurrence relation for e_n = the number of edges of the graph Q_n

d) Give a recurrence relation for e_n = the number of edges of the graph W_n

9. (5%)

If $\mathbf{M}_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$, determine if R is: (a) reflexive (b) symmetric (c) antisymmetric (d) transitive.

10. In the questions below give an example or else prove that there are none. (5%)

a) A relation on {a,b,c} that is reflexive and transitive, but not antisymmetric.

b) A relation on {1,2} that is symmetric and transitive, but not reflexive.

c) A relation on {1,2,3} that is reflexive and transitive, but not symmetric.

11. If R = {(1,2),(1,4),(2,3),(3,1),(4,2)}, (5%)

a) Find the reflexive closure of R.

b) Find the Symmetric closure of R.

12. In the questions below, describe each sequence recursively. Include initial conditions and assume that the sequences begin with a_1. (10%)

a) a_n = the number of bit strings of length n with an even number of 0s.

b) a_n = the number of bit strings of length n that begin with 1.

c) a_n = the number of bit strings of length n that contain a pair of consecutive 0s.

d) a_n = the number of ways to go down an n-step staircase if you go down 1,2, or 3 steps at a time.

13. In the question below write the first seven terms of the sequence determined by the generating function. (5%)

a) $(x+3)^2$　b) $(1+x)^5$　c) $1/(1-3x)$　d) $X^2/(1-x)$

14. In the questions below find a closed form for the generating function for the sequence.

a) 4,8,16,32,64,.....

b) 1,0,1,0,1,0,1,0,1,0,....

c) 2,0,0,2,0,0,2,0,0,....

d) 2,4,6,8,10,..... (10%)

15. Set up a generating function and use it to find the number of ways in which eleven identical coins can be put in three distinct envelopes if each envelope has at least two but no more than five coins in it. (5%)

16. A market sells ten kinds of soda. You want to buy 12 bottles. How many possibilities are there? if you want at most three bottles of any kind? (5%)

17. Determine whether $(p \rightarrow q) \wedge (\neg p \rightarrow q) \equiv q$. (5%)