元智大學 101 學年度研究所 **碩士班** 招生試題卷

系(所)別：	工業工程與管理學系碩士班	組別：	不分組	科目：	作業研究	用紙第 1 頁共 2 頁

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1. (30%) Consider the following linear programming problem:

Maximize $Z = -2x_1 + 3x_2 + 4x_3$

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\begin{aligned}
\text{s.t.}\quad -2x_1 + x_2 + x_3 &\ge 4 \\
x_1 + x_2 + x_3 &= 5 \\
x_1 \qquad\quad + x_3 &\le 3 \\
x_1, x_2, x_3 &\ge 0
\end{aligned}
$$

(a) (5%) Write down the dual of this primal problem.

(b) (5%) It is given that the primal optimal solution is $(x_1, x_2, x_3) = (0, 2, 3)$. What is the corresponding optimal basis when you apply the simplex method?

(c) (5%) Calculate the dual optimal solution.

(d) (5%) For the primal problem, what is the range of the third right-hand-side value b_3 (currently 3) such that the current optimal basis (or dual optimal solution) remains unchanged?

(e) (5%) For the primal problem, what is the range of profit c_2 for x_2 (currently $c_2 = 3$) such that the current optimal solution $(x_1, x_2, x_3) = (0, 2, 3)$ remains unchanged?

(f) (5%) Suppose that a new variable x_4 with coefficient vector $(c_4, a_{14}, a_{24}, a_{34})^T = (5, 2, 1, 3)$ is introduced into the linear programming model. Will the optimal solution change? Why?

2. (20%) Consider the following profit maximization transportation problem. The number inside the rectangle represents the profit earned by shipping one unit via its corresponding path. For example, the profit of shipping one unit from supply node 1 to demand node B is 12 dollars.

(a) (5%) Use maximum profit rule to find a basic feasible solution (BFS). Note that at each step, this rule selects a cell where profit is the highest among the candidates.

(b) (5%) Find an optimal BFS using the BFS found in (a).

(c) (5%) What is the range of profit P_{3B} (currently 10) such that the current optimal solution remains unchanged?

(d) (5%) What will be the new optimal BFS if node 1 increases its supply from 20 to 25, and node A increases its demand from 30 to 35?

	A	B	C	D	supply
1	11	12	9	6	20
2	13	15	8	4	40
3	7	10	5	8	60
demand	30	30	30	30	

3. (25%) Consider a single-server queueing system where customers arrive according to a Poisson process with rate $\lambda = 2$ per hour. The system has unlimited queue length. Customers can be classified into two types: type I customers require an exponential service time with rate

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$\mu = 4$ per hour, and type II customers require an exponential service time with rate $\mu = 2$ per hour. The probability that an arriving customer is type I is p, and type II is 1 – p.

Suppose that p = 1.0, i.e., the system will only contain type I customers. Answer questions (a) and (b).

(a) (5%) What is the steady-state probability that the system has two customers?

(b) (5%) In the long run, what is the average number of customers in the system?

Now suppose that p = 0.5. This problem can then be viewed as an M/G/1/∞ model, where the service time is a hyper-exponential distribution. The expected number of customers in the queue when the system reaches steady-state is $L_q = (\lambda^2 \cdot \sigma^2 + \rho^2)/2(1-\rho)$, where σ^2 is the variance of the service time and $\rho = \lambda/\mu$. Answer questions (c) to (e).

(c) (5%) Compute the mean and the standard deviation for the hyper-exponential distribution.

(d) (6%) What are L_q, and W? (W is the expected waiting time of a customer in the system)

(e) (4%) What is the expected number of type II customers in the system?

4. (25%) Consider an inventory problem for a camera store. This store stocks a particular type of camera that can be ordered weekly. Let X_k represent the number of cameras on hand at the end of week k (i.e. inventory level of week k). The store owner, Johnson, adopts the following ordering policy: If $X_k \leq 1$, order two cameras; otherwise, do not order any cameras. It is assumed that every order is placed at weekend and delivered on the following Monday morning before the store opens. Suppose that the weekly demand for the camera, $\{D_k: k = 1,\ldots\}$, are identical and independent random variables with the following probability distribution: $\Pr\{D_k = 0\} = 0.2$, $\Pr\{D_k = 1\} = 0.3$, $\Pr\{D_k = 2\} = 0.3$, $\Pr\{D_k = 3\} = 0.1$, $\Pr\{D_k = 4\} = 0.1$. Answer the following questions.

(a) (5%) Explain why the stochastic process $\{X_k, k = 0, 1, \ldots\}$ is a Markov chain, and construct the one-step transition probability matrix.

(b) (5%) Compute the probability, $\Pr\{X_4 = 1 \mid X_2 = 3\}$.

(c) (5%) Explain why the steady-state probabilities exist for this Markov chain, and list a set of equations for computing the steady-state probabilities $\{\pi_j\}$. (Do not solve)

(d) (5%) What is the average time length between two consecutive camera orders? (You can use the notation in (c) to answer this question).

(e) (5%) List a set of equations that can compute the expected first passage time from state 1 to state 0. (Do not solve)