

●不可使用電子計算機

1. 按題目順序作答並且題號標示清楚，否則不予評分。
2. 每題答案應有合理的步驟說明，否則不給分。

1. (15 pts.) State *three axioms of probability* in detail and prove the following equation by the axioms of probability.

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

where E and F are two events defined on a sample space S .

2. (10 pts.) Suppose that three coins, NT\$5, NT\$10, and NT\$50 are placed into three identical envelopes. Three envelopes are then randomly given the designations A , B and C . The coins in the envelopes A and B are compared. If the smaller value of these two coins is then compared with the coin in the envelope C , what is the probability that it is also smaller than the coin in the envelope C ?

3. (5+5+8+7 pts.) The joint probability density function of X and Y is given by

$$f(x, y) = \begin{cases} k(x^2 + xy), & 0 < x < 1, 0 < y < 2 \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Find k .
- (b) Find $P(X > Y)$.
- (c) Find the density function of the random variable $Z = X - Y$.
- (d) Verify that X and Y are dependent.

4. (15 pts.) State the *Central Limit Theorem, CLT* in detail and explain the theorem by an example. Why is the *CLT* important in industry?

●不可使用電子計算機

5. (10+10+5+5+5 pts.) Let p denote the defective rate of some production line (the *population parameter* of interest). Suppose we randomly inspected 20 items produced by the production line and found 2 defectives (the *observed sample*).

- (a) Explain how to construct a confidence interval (L, U) of p based on the observed sample such that $P(p \in (L, U)) = 95\%$. (Note: The *CLT* is not applicable here.)
- (b) Explain how to construct the rejection region based on the observed sample, if we would like to test $H_0 : p = 5\%$ against $H_1 : p > 5\%$? Test at α level.
- (c) In question (b), find the p -value. (Express only, final calculation is unnecessary.)
- (d) Explain the meaning of p -value. (Note: Answer is NOT the definition of the p -value.)
- (e) Why the test decision can be determined by p -value?