# 淡江大學 101 學年度碩士班招生考試試題

系別：資訊工程學系
資訊工程學系資訊網路與通訊碩士班

科目：數　學(含離散數學、線性代數)

考試日期：2 月 26 日(星期日) 第 3 節　　本試題共 5 大題， 1 頁

1. Fill in the blanks or answer true/false (填充題或是非題) 4pts each. 48 pts

________(1). Find the coefficient of $x^6$ in the expansion of $(2 - x^3)^5$.

________(2). Find the number of permutations of 1234 that leave 3 in the third position but leave no other integer in its own position.

________(3). Suppose R and S are relations on{a, b, c, d}, where R={(a,b), (a,d), (b,c), (c, c), (d,a)} and S={(a,c), (b,d), (d,a)}. Find RoS.

________(4). If the truth value for " $p \to q$" is **false** then the truth value of its **converse** " $q \to p$" must be **true**.

________(5). Find the value of $\left\lceil \frac{1}{2} \cdot \left\lceil -\frac{23}{4} \right\rceil + \left\lfloor \frac{3}{2} \right\rfloor \right\rceil$.

________(6). If $\boldsymbol{T}$ is a full binary tree of height 4, let $x$ be the minimum number of leaves in $\boldsymbol{T}$ and $y$ be the maximum number of leaves in $\boldsymbol{T}$. Find the value of $x + y$.

________(7). Find the value of the postfix expression 8 2 3 * − 4 ↑ 9 3 / +

________(8). Find the area of the triangle determined by the vectors $u = (5,3)$, $v = (-1,5)$.

________(9). Let $A = \begin{bmatrix} 1 & 0 \\ -5 & 2 \end{bmatrix}$, then there exist elementary matrices $E_1$ and $E_2$ such that $A = E_1E_2$.

________(10). $(2, 2, 4) \times (-2, 3, -3)$ is parallel to $(9, -1, -5)$.

________(11). The line $x = 1+5t$, $y = 1-2t$, $z = 4+t$ and the plane $2x + 3y - 4z = 1$ are perpendicular.

________(12). If $S_1$ and $S_2$ are two linearly dependent sets of vectors, then so is the union $S_1 \cup S_2$.

**Show enough works to get full credits for problems 2 ~ 5. Answer alone gets at most half credit.**

2. Let $u = (2, -1, 3)$ and $a = (4, -1, 2)$. Find (i) the vector component of $u$ along $a$; (ii) the vector component of $u$ orthogonal to $a$. (16 pts)

3. Find the least squares solution of the linear system $Ax = b$ where (12 pts)

$$A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ -1 & 2 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 7 \\ 0 \\ -7 \end{bmatrix}.$$

4. Show that $1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \cdots + n \cdot n! = (n+1)! - 1$ for $n = 1, 2, 3, \ldots$ ( 12 pts)

5. Find the number of ways to distribute 7 different toys to 4 children such that the youngest one must get the red car and every child gets at least one toy. (12 pts)