淡江大學 101 學年度碩士班招生考試試題

系別：數學學系　　科目：微積分 60%及線性代數 40%

考試日期：2 月 26 日(星期日) 第 2 節　　本試題共 6 大題， 1 頁

1. **Find the integrals:**

(1) $\int \frac{e^{-1/x}}{x^2} dx$　　(2) $\int \frac{5}{x^2 - 6x + 25} dx$　　(20%)

2. **Evaluate** $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n+i}$. **(10%)**

3. Determine convergence or divergence for each of the series. Indicate the test you used.

(a) $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$ **(b)** $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ **(c)** $\sum_{n=1}^{\infty} (\frac{n}{3n+2})^n$ **(d)** $\sum_{n=1}^{\infty} \frac{3n+1}{n^3 - 4}$

(e) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{2^n}$ **(30 %)**

4. Find a basis of P_2 **from** $\{1+2x^2,\ x+x^2,\ 2-x+3x^2,\ 3+x+7x^2, x^2\}$,

where $P_2 = \{ax^2 + bx + c \,|\, a, b, c \in R\}$. **(10%)**

5. Let $A = \begin{bmatrix} 1 & -2 & 1 & 1 \\ -1 & 2 & 0 & 1 \\ 2 & -4 & 1 & 0 \end{bmatrix}$, **(20 %)**

(a) Find a basis for the subspace of the solutions of $A\vec{X} = \vec{0}$.

(b) Find a basis for the row space $R(A)$ **of** A.

(c) Find a basis for the column space $C(A)$ **of** A..

(d) Find rank(A).

6. Let $T : V \to W$ **be any linear transformation, prove that** T **is 1-1 if and only if ker(**T**) =** $\{\vec{0}\}$**, where** V **and** W **are vector spaces.** **(10%)**