

淡江大學 101 學年度碩士班招生考試試題

系別：數學學系

科目：統計學

考試日期：2月26日(星期日) 第3節

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1. (20%) The random variable X takes the values 0, 1, 2 according to one of the following distributions:

	$P(X = 0)$	$P(X = 1)$	$P(X = 2)$	
Distribution 1	p	$3p$	$1 - 4p$	$0 < p < 1/4$
Distribution 2	p	p^2	$1 - p - p^2$	$0 < p < 1/2$

In each case determine whether the family of distribution of X is complete?

2. (20%) Let X_1, \dots, X_n be a random sample of size n , $n > 1$, from a distribution with probability density function

$$f(x; \theta_1, \theta_2) = \frac{1}{\theta_2} \exp\left(-\frac{x - \theta_1}{\theta_2}\right), \quad x \geq \theta_1, \theta = (\theta_1, \theta_2)' \in \Omega = \mathbb{R} \times (0, \infty).$$

Find the maximum likelihood estimators of θ_1 and θ_2 .

3. (30%) Let X_1 and X_2 be a random sample of size 2 from $N(\theta, 1)$, $-\infty < \theta < \infty$.
Let

$$Y = X_1 + X_2, Z = X_1 + 2X_2, W = \left(\frac{X_1 + X_2}{2}\right)^2, V = \left(\frac{X_1 + X_2}{2}\right)^3.$$

- a) (10%) Which of the 4 statistics are sufficient for θ ?
- b) (10%) Show that the condition distribution of Z , given $Y = y$, does not depend on θ .
- c) (10%) Find $E(Z|Y = y) = d(y)$ and the variance of $d(Y)$.

4. (10%) Let X_1, \dots, X_n be i.i.d. random variables from Uniform $(0, \theta)$. Let

$$X_{(n)} = \max_{1 \leq i \leq n} X_i.$$

Use $X_{(n)}/\theta$ to construct the shortest confidence interval for θ with confidence coefficient $1 - \alpha$.

5. (20%) Suppose X is one observation from a distribution with probability density function

$$f(x; \theta) = \theta x^{\theta-1}, 0 < x < 1, \theta > 0.$$

- a) (10%) Find the most powerful level α test of $H_0 : \theta = 1$ versus $H_1 : \theta = 2$.
- b) (10%) Is there a UMP test of $H_0 : \theta \leq 1$ versus $H_1 : \theta > 1$?