# 淡江大學 101 學年度碩士班招生考試試題

系別：物理學系　　科目：近代物理

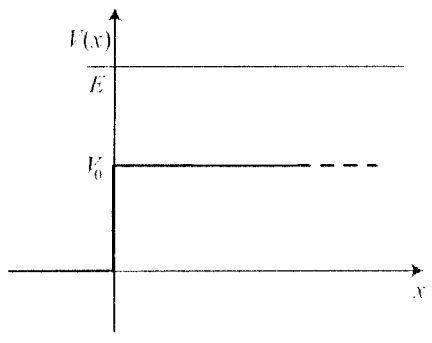
考試日期：2 月 26 日 (星期日) 第 2 節　　本試題共 5 大題，1 頁

1. (20 points) Discuss the following experiments and their significance.
   (a) Rutherford scattering experiment
   (b) Frank-Hertz experiment
   (c) Stern-Gerlach experiment
   (d) Compton scattering experiment

2. (20 points) A particle of rest mass $m_1$ moves with relativistic speed $v_1$ along the $x$-axis, in the positive direction. It collides with a particle of rest mass $m_2$, which is at rest. The two stick together, and continue to move as one particle.
   (a) What is the total momentum $p$ of the system before the collision?
   (b) What is the total energy $E$ of the system before the collision?
   (c) What is the velocity $v$ of the final particle?
   (d) What is the rest mass $m$ of the final particle?

3. (20 points) Consider a hypothetical three-state quantum system that possesses three orthonormal states, denoted by $|1\rangle$, $|2\rangle$ and $|3\rangle$. The states $|\psi_1\rangle$ and $|\psi_2\rangle$ are given by
$$|\psi_1\rangle = \frac{1}{2}|1\rangle + \frac{i}{2}|2\rangle + \frac{1}{\sqrt{2}}|3\rangle \quad \text{and} \quad |\psi_2\rangle = 2|1\rangle + i|2\rangle.$$
   (a) Are these states normalized? If not, find the normalized states.
   (b) What is the scalar product $\langle\psi_1|\psi_2\rangle$ for the *normalized* states?
   (c) What is the probability of finding $|\psi_1\rangle$ in $|2\rangle$?
   (d) What is the probability of finding $|\psi_2\rangle$ in $|3\rangle$?

4. (20 points) Consider a one-dimensional particle of mass $m$ and energy $E$ that moves *from left to right* and encounters the potential barrier depicted in the figure below with $0 < V_0 < E$.
   (a) Find the wave function of the particle.
   (b) Find the transmission coefficient.




5. (20 points) The radial and spherical parts of the wave function for an electron of a hydrogen atom in the $2p$ state are given by $R_{2,1}(r) = A\, r\, e^{-r/2a_0}$ and $Y_{1,1}(\theta,\phi) = B \sin\theta\, e^{i\phi}$, where $A$ and $B$ are constants and $a_0$ is the Bohr radius.
   (a) Determine the value of $B$.
   (b) What are the expectation values of $L_z$ and $L^2$?
   (c) What are the radial and spherical probability densities of finding the electron?
   (d) Find the radius at which the electron is most likely to be found.

Note: $\displaystyle\int_{-\infty}^{\infty} e^{-ax^2}dx = \sqrt{\frac{\pi}{a}}$ and $\displaystyle\int_{-\infty}^{\infty} x^2 e^{-ax^2}dx = \frac{1}{2a}\sqrt{\frac{\pi}{a}}$ $(a > 0)$.