

淡江大學 101 學年度碩士班招生考試試題

系別：管理科學學系

科目：統計學

考試日期：2月26日(星期日) 第3節

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本試題雙面印刷

1. Suppose the length of time an electric bulb lasts, X , is a random variable with cumulative

$$\text{Distribution } F(x) = \begin{cases} 0 & , x < 0 \\ 1 - e^{-x/500} & , x \geq 0 \end{cases}$$

Find (a) the probability that the bulb lasts beyond 300 hours. (5 points)

(b) the p.d.f. $f(x)$ of X . (5 points)

2. Let X and Y be jointly distributed with density function $f(x, y) = \begin{cases} 1 & , 0 < x < 1, 0 < y < 1 \\ 0 & , \text{ otherwise} \end{cases}$

Find $F(\lambda|X > Y) = P(X \leq \lambda|X > Y)$. (10 points)

3. Let X be distributed with density function $f(x) = \begin{cases} \lambda e^{-\lambda x} & , \text{ for } x > 0 \\ 0 & , \text{ otherwise} \end{cases}$

If Y is a new random variable defined as $Y = \ln X$, find the density function of Y . (10 points)

4.(a) Find the moment generating function of a standard normal random variable. (6 points)

(b) Use the result of the part (a) to calculate the variance of this variable. (4 points)

5. Let X be uniformly distributed, $f(x, \theta) = \begin{cases} \frac{1}{\theta} & , 0 < x \leq \theta, 0 < \theta < \infty \\ 0 & , \text{ otherwise} \end{cases}$

Find the maximum likelihood estimate for the parameter θ . (10 points)

6. Let $f(x, \theta) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-\theta)^2}$ be a probability density function. Assuming $H_0: \theta = \theta_0$, and

$H_1: \theta = \theta_1$ where $\theta_0 > \theta_1$, find the best critical region, C , for this test. (10 points)

7. Given the following pairs of measurements for the two variables:

X 5 8 3 9 10 12

Y 9 12 5 15 18 20

(a) What is the correlation between X and Y ? (5 points)

(b) Calculate the regression line $Y = aX + b$. (5 points)

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8. The following data was gathered in an experiment comparing the effects of three insecticides in controlling a certain species of parasitic beetles. Each observation represents the number of each insects found dead in a certain fixed area containing the insecticides.

Insecticide	n_j	Y_{ij}	$\bar{Y}_{.j}$	$\sum_{i=1}^{n_j} (Y_{ij} - \bar{Y}_{.j})^2$
1	4	11, 9, 13, 11	11	8
2	6	25, 28, 31, 27, 30, 33	29	42
3	5	19, 23, 19, 21, 20	20.4	11.2

Set up the ANOVA table with test statistic but without conclusion. (10 points)

9. A die was tossed 120 times and the results are listed below.

Upterned face	1	2	3	4	5	6
Frequency	18	23	16	21	18	24

Compute the χ^2 statistic for this 1 by 6 contingency table under the hypothesis that the die was fair and the level of significance $\alpha = 0.05$ and make your conclusion (provide these two values $\chi^2_{0.05, 5} = 11.071$, $\chi^2_{0.05, 6} = 12.592$ for the critical value). (10 points)

10. Consider the following data obtained from testing the breaking strength of ceramic tile manufactured by a new cheaper process: 20, 42, 18, 21, 22, 35, 19, 18, 26, 20, 21, 32, 22, 20, 24. Suppose that experience with the old process produced a median of 25. Then test the hypothesis

$H_0: M = 25$ against $H_1: M < 25$; and provide the critical value $z_{0.05} = 1.645$

Use the Sign Test to make your conclusion. (10 points)