

# 淡江大學 101 學年度碩士班招生考試試題

系別：保險學系

科目：統計學

考試日期：2月26日(星期日) 第2節

本試題共二大題，二頁

本試題雙面印刷

第一大題(含 10 小題，每小題 7 分，共 70 分)

1. Let  $A$  and  $B$  be any two events defined on  $S$ ,  $A^c$  and  $B^c$  represent the complement of  $A$  and  $B$  respectively. If  $P(A) = \frac{1}{2}$  (probability of  $A$  is  $\frac{1}{2}$ ),  $P(B) = \frac{1}{3}$ , and  $P(A \cup B) = \frac{3}{4}$ , find (1)  $P(A \cap B)$  (2)  $P(A^c \cap B^c)$  (3)  $P(A \cap B^c)$ .

2. In actuarial science, one of the models used for describing mortality is

$$f(t) = k \cdot t^2(100 - t)^2, \quad 0 \leq t \leq 100$$

where  $t$  denotes the age at which a person dies.

(4) Find  $k$ .

(5) Let  $A$  be the event "Person lives past 50". Find  $P(A)$ .

(6) Find the probability that a person will die between the ages of 80 and 85 given that that person has lived to be at least 70.

3. Consider an experiment of tossing a single die, let the events  $A, B$  and  $C$  be the following

$A$ : Observe an even number.

$B$ : Observe an odd number.

$C$ : Observe a 1 or 2.

(7) Explain and determine whether  $A$  and  $B$  are independent events.

(8) Explain and determine whether  $A$  and  $C$  are independent events.

4. Consider the following PDF of two continuous variables  $X$  and  $Y$ ,

$$f(x, y) = 2 + k(x + y); \quad 0 \leq x \leq 1, 0 \leq y \leq 1$$

(9) Find the marginal PDF of  $Y$ .

(10) Are  $X$  and  $Y$  statistically independent? Why?

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第二大題(含6小題，每小題5分，共30分)

5. The following table gives the joint PDF of the discrete variables X and Y.

X	Y	
	3	6
-2	0.27	0
0	0.08	0.04
2	0.16	0.10
3	0	0.35

This table tells us that the probability that X takes the value of -2 while Y simultaneously takes the value of 3 is 0.27, and so on.

- (1) Find the conditional probabilities  $f(X = 0 | Y = 6)$  and  $f(Y = 3 | X = 2)$ .
- (2) If  $Z = (X - 1)^2$ , find the expected value of Z,  $E(Z)$ .
- (3) If  $W = (X - Y)^2$ , find the expected value of W,  $E(W)$ .
- (4) Compute the variance of  $2X - Y$ ,  $\text{var}(2X - Y)$ .
- (5) Compute the expected value of Y given that X is 2,  $E(Y | X = 2)$ .
- (6) Compute the variance of Y given that X is 2,  $\text{var}(Y | X = 2)$ .