

科目：統計學

系所組：金融與國際企業學系

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1. Let the joint probability function of X_1 and X_2 be

$$f(x_1, x_2) = \begin{cases} \frac{1}{18} x_1 x_2, & x_1 = 1, 2, 3, x_2 = 1, 2 \\ 0, & \text{elsewhere} \end{cases}$$

- (5%) Find the marginal probability function of X_1 random variable.
 - (5%) Find the probability of $\Pr(X_1 \geq 2)$.
 - (5%) Find the conditional mean of X_1 , given $X_2=1$.
 - (5%) Find the conditional variance of X_1 , given $X_2=1$.
2. (10%) Let X and Y are continuous random variables. It will be assumed that σ_x^2 , σ_y^2 and $\rho\sigma_x\sigma_y$, the variances of X and Y and the covariance of X and Y are not zero. And also assumed that μ_x and μ_y , the unconditional mean of X and Y are not zero. If the conditional mean of Y , given $X=x$ is a linear form, say $a + bx$, where a and b are constants. Find the constants of a and b .
3. Let X_1, X_2, \dots, X_n denote a random independent sample of size $n \geq 2$ from a normal distribution that is $N(\mu, \sigma^2)$. It is well known that a linear combination of normal random variables is a normal random variable, and zero covariance of normal random variables can imply the independent property of two normal random variables. Further, a squared standard normal random variable is a chi-square random variable with the degree of freedom one. Show that
- (5%) the sample mean \bar{X} is $N(\mu, \frac{\sigma^2}{n})$.
 - (5%) the $\frac{(n-1)S^2}{\sigma^2}$ is chi-square random variable with the degree of freedom $n-1$, where S^2 is the sample variance.
 - (5%) the sample mean \bar{X} and the sample variance S^2 are independent.
4. (10%) Find the exponential distribution from the Poisson distribution.
5. (5%) Show that $U = F(X)$ is a uniform distribution, where $F(\cdot)$ is the invertible cumulative distribution function of X random variable.
6. (10%) Assumed that the random samples are independently selected from two populations and that the populations are normally distributed and have equal unknown variances. Please construct the confidence interval estimate for the difference in the means of two independent populations.
7. (10%) Write down the procedures of Levene Test for homogeneity of variance.
8. (20%) The four assumptions of regression are known by the acronym LINE, those are linearity, independence of errors, normality of error, and equal variance. What procedures can help you to determine whether the regression model that has been selected is appropriate.

- ※ 注意： 1. 考生須在「彌封答案卷」上作答。
 2. 本試題紙空白部份可當稿紙使用。
 3. 考生於作答時可否使用計算機、法典、字典或其他資料或工具，以簡章之規定為準。