

科目：統計學

系所別：統計資訊學系應用統計研究所碩士班

2 附卷 12 頁

註：題目共有 5 大題，附表包括：標準常態機率表，卡方分配表，F 分配表，t 分配表，Poisson 分配機率表。

1. (20%) Large percent changes in a common stock price index on any single day are rare. Consider a change in the index (either up or down) of 10 percent or more on a single day to be an "occurrence." Assume that these occurrences follow a Poisson process with average of 0.3 per year.
 - (1) What is the nature of the probability distribution for the lengths of time between successive occurrences for this index?
 - (2) What is the probability that an occurrence is followed by the next occurrence within six months?
2. (20%) The 20th and 60th percentiles of a normal random variable are -4.3 and 15.8, respectively. Find the mean and the standard deviation of the random variable.
3. (20%) A lathe operator is to be observed at random times to estimate the proportion of times the operator is in a productive state. It is desired to estimate this process proportion within ± 0.04 , with a 99 percent confidence interval. It is expected that the process proportion is 0.9 or greater.
 - (1) What sample size will be needed to ensure the estimate of the desired precision?
 - (2) It was finally decided to take $n=200$ random observations. The lathe operator was found to be in a productive state in 176 of these observations. Construct a 99 percent confidence interval for the process proportion. Interpret results.
4. (25%) A rehabilitation center researcher was interested in examining the relationship between physical fitness prior to surgery of persons undergoing corrective knee surgery and time required in physical therapy until successful rehabilitation. Patient records in the rehabilitation center were examined, and 24 male subjects ranging in age from 18 to 30 years who had undergone similar corrective knee surgery during the past year were selected for the study. The number of days required for successful completion of physical therapy and the prior physical fitness status (below average, average, above average) for each patient as follow:

| | | | | | | | | | | |
|------------------------|----|----|----|----|----|----|----|----|----|----|
| Fitness status\patient | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| below average | 29 | 42 | 38 | 40 | 43 | 40 | 30 | 42 | | |
| average | 30 | 35 | 39 | 28 | 31 | 29 | 35 | 29 | 33 | 31 |
| above average | 26 | 32 | 21 | 20 | 23 | 22 | | | | |

※ 注意：1. 考生須在「彌封答案卷」上作答。
 2. 本試題紙空白部份可當稿紙使用。
 3. 考生於作答時可否使用計算機、法典、字典或其他資料或工具，以簡章之規定為準。

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(continued)

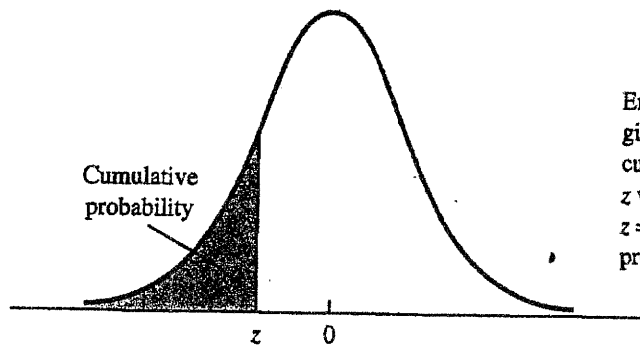
- (1) Test for all pairs of factor level means whether or not they differ. Conclude with the level of significance 0.05.
 - (2) What is the nature of the relationship between physical fitness status and time of required physical therapy?
 - (3) Define two-factor analysis of variance model with assumptions. Is the above problem a case of the two-factor analysis of variance? Why or why not? Use an example to explain if necessary.
5. (15%) Define (1) Random Variable. (2) Random Sample.

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2. 本試題紙空白部份可當稿紙使用。

3. 考生於作答時可否使用計算機、法典、字典或其他資料或工具，以簡章之規定為準。

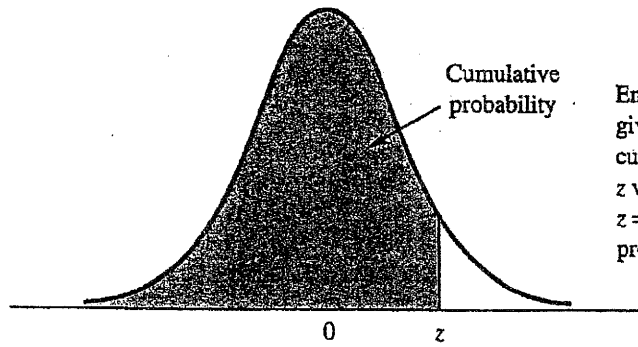
TABLE 1 CUMULATIVE PROBABILITIES FOR THE STANDARD NORMAL DISTRIBUTION



Entries in the table give the area under the curve to the left of the z value. For example, for $z = -.85$, the cumulative probability is .1977.

| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| -3.0 | .0013 | .0013 | .0013 | .0012 | .0012 | .0011 | .0011 | .0011 | .0010 | .0010 |
| -2.9 | .0019 | .0018 | .0018 | .0017 | .0016 | .0016 | .0015 | .0015 | .0014 | .0014 |
| -2.8 | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| -2.7 | .0035 | .0034 | .0033 | .0032 | .0031 | .0030 | .0029 | .0028 | .0027 | .0026 |
| -2.6 | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |
| -2.5 | .0062 | .0060 | .0059 | .0057 | .0055 | .0054 | .0052 | .0051 | .0049 | .0048 |
| -2.4 | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |
| -2.3 | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |
| -2.2 | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| -2.1 | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| -2.0 | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| -1.9 | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| -1.8 | .0359 | .0351 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| -1.7 | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| -1.6 | .0548 | .0537 | .0526 | .0516 | .0505 | .0495 | .0485 | .0475 | .0465 | .0455 |
| -1.5 | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| -1.4 | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0721 | .0708 | .0694 | .0681 |
| -1.3 | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| -1.2 | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| -1.1 | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |
| -1.0 | .1587 | .1562 | .1539 | .1515 | .1492 | .1469 | .1446 | .1423 | .1401 | .1379 |
| -.9 | .1841 | .1814 | .1788 | .1762 | .1736 | .1711 | .1685 | .1660 | .1635 | .1611 |
| -.8 | .2119 | .2090 | .2061 | .2033 | .2005 | .1977 | .1949 | .1922 | .1894 | .1867 |
| -.7 | .2420 | .2389 | .2358 | .2327 | .2296 | .2266 | .2236 | .2206 | .2177 | .2148 |
| -.6 | .2743 | .2709 | .2676 | .2643 | .2611 | .2578 | .2546 | .2514 | .2483 | .2451 |
| -.5 | .3085 | .3050 | .3015 | .2981 | .2946 | .2912 | .2877 | .2843 | .2810 | .2776 |
| -.4 | .3446 | .3409 | .3372 | .3336 | .3300 | .3264 | .3228 | .3192 | .3156 | .3121 |
| -.3 | .3821 | .3783 | .3745 | .3707 | .3669 | .3632 | .3594 | .3557 | .3520 | .3483 |
| -.2 | .4207 | .4168 | .4129 | .4090 | .4052 | .4013 | .3974 | .3936 | .3897 | .3859 |
| -.1 | .4602 | .4562 | .4522 | .4483 | .4443 | .4404 | .4364 | .4325 | .4286 | .4247 |
| -.0 | .5000 | .4960 | .4920 | .4880 | .4840 | .4801 | .4761 | .4721 | .4681 | .4641 |

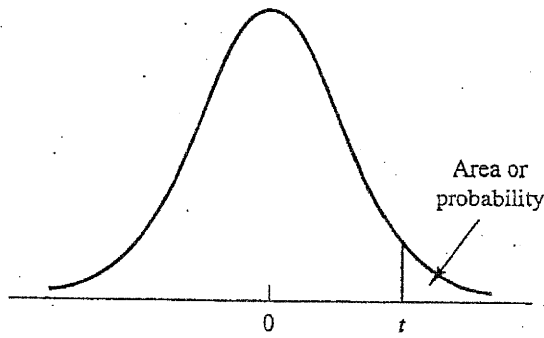
TABLE 1 CUMULATIVE PROBABILITIES FOR THE STANDARD NORMAL DISTRIBUTION (Continued)



Entries in the table give the area under the curve to the left of the z value. For example, for $z = 1.25$, the cumulative probability is .8944.

| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| .0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| .1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| .2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| .3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| .4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| .5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| .6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| .7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| .8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| .9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9913 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9986 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |

TABLE 2 *t* DISTRIBUTION



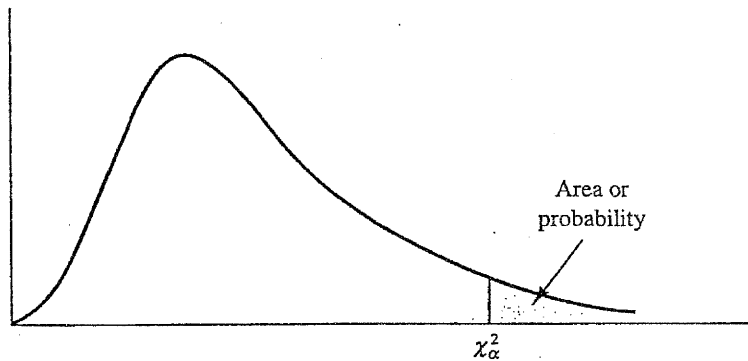
Entries in the table give *t* values for an area or probability in the upper tail of the *t* distribution. For example, with 10 degrees of freedom and a .05 area in the upper tail, $t_{.05} = 1.812$.

| Degrees of Freedom | Area in Upper Tail | | | | | |
|--------------------|--------------------|-------|-------|--------|--------|--------|
| | .20 | .10 | .05 | .025 | .01 | .005 |
| 1 | 1.376 | 3.078 | 6.314 | 12.706 | 31.821 | 63.656 |
| 2 | 1.061 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 |
| 3 | .978 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 |
| 4 | .941 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 |
| 5 | .920 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 |
| 6 | .906 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 |
| 7 | .896 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 |
| 8 | .889 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 |
| 9 | .883 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 |
| 10 | .879 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 |
| 11 | .876 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 |
| 12 | .873 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 |
| 13 | .870 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 |
| 14 | .868 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 |
| 15 | .866 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 |
| 16 | .865 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 |
| 17 | .863 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 |
| 18 | .862 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 |
| 19 | .861 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 |
| 20 | .860 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 |
| 21 | .859 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 |
| 22 | .858 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 |
| 23 | .858 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 |
| 24 | .857 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 |
| 25 | .856 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 |
| 26 | .856 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 |
| 27 | .855 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 |
| 28 | .855 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 |
| 29 | .854 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 |
| 30 | .854 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 |
| 31 | .853 | 1.309 | 1.696 | 2.040 | 2.453 | 2.744 |
| 32 | .853 | 1.309 | 1.694 | 2.037 | 2.449 | 2.738 |
| 33 | .853 | 1.308 | 1.692 | 2.035 | 2.445 | 2.733 |
| 34 | .852 | 1.307 | 1.691 | 2.032 | 2.441 | 2.728 |

TABLE 2 *t* DISTRIBUTION (Continued)

| Degrees of Freedom | Area in Upper Tail | | | | | |
|-----------------------|--------------------|-------|-------|-------|-------|-------|
| | .20 | .10 | .05 | .025 | .01 | .005 |
| 35 | .852 | 1.306 | 1.690 | 2.030 | 2.438 | 2.724 |
| 36 | .852 | 1.306 | 1.688 | 2.028 | 2.434 | 2.719 |
| 37 | .851 | 1.305 | 1.687 | 2.026 | 2.431 | 2.715 |
| 38 | .851 | 1.304 | 1.686 | 2.024 | 2.429 | 2.712 |
| 39 | .851 | 1.304 | 1.685 | 2.023 | 2.426 | 2.708 |
| 40 | .851 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 |
| 41 | .850 | 1.303 | 1.683 | 2.020 | 2.421 | 2.701 |
| 42 | .850 | 1.302 | 1.682 | 2.018 | 2.418 | 2.698 |
| 43 | .850 | 1.302 | 1.681 | 2.017 | 2.416 | 2.695 |
| 44 | .850 | 1.301 | 1.680 | 2.015 | 2.414 | 2.692 |
| 45 | .850 | 1.301 | 1.679 | 2.014 | 2.412 | 2.690 |
| 46 | .850 | 1.300 | 1.679 | 2.013 | 2.410 | 2.687 |
| 47 | .849 | 1.300 | 1.678 | 2.012 | 2.408 | 2.685 |
| 48 | .849 | 1.299 | 1.677 | 2.011 | 2.407 | 2.682 |
| 49 | .849 | 1.299 | 1.677 | 2.010 | 2.405 | 2.680 |
| 50 | .849 | 1.299 | 1.676 | 2.009 | 2.403 | 2.678 |
| 51 | .849 | 1.298 | 1.675 | 2.008 | 2.402 | 2.676 |
| 52 | .849 | 1.298 | 1.675 | 2.007 | 2.400 | 2.674 |
| 53 | .848 | 1.298 | 1.674 | 2.006 | 2.399 | 2.672 |
| 54 | .848 | 1.297 | 1.674 | 2.005 | 2.397 | 2.670 |
| 55 | .848 | 1.297 | 1.673 | 2.004 | 2.396 | 2.668 |
| 56 | .848 | 1.297 | 1.673 | 2.003 | 2.395 | 2.667 |
| 57 | .848 | 1.297 | 1.672 | 2.002 | 2.394 | 2.665 |
| 58 | .848 | 1.296 | 1.672 | 2.002 | 2.392 | 2.663 |
| 59 | .848 | 1.296 | 1.671 | 2.001 | 2.391 | 2.662 |
| 60 | .848 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 |
| 61 | .848 | 1.296 | 1.670 | 2.000 | 2.389 | 2.659 |
| 62 | .847 | 1.295 | 1.670 | 1.999 | 2.388 | 2.657 |
| 63 | .847 | 1.295 | 1.669 | 1.998 | 2.387 | 2.656 |
| 64 | .847 | 1.295 | 1.669 | 1.998 | 2.386 | 2.655 |
| 65 | .847 | 1.295 | 1.669 | 1.997 | 2.385 | 2.654 |
| 66 | .847 | 1.295 | 1.668 | 1.997 | 2.384 | 2.652 |
| 67 | .847 | 1.294 | 1.668 | 1.996 | 2.383 | 2.651 |
| 68 | .847 | 1.294 | 1.668 | 1.995 | 2.382 | 2.650 |
| 69 | .847 | 1.294 | 1.667 | 1.995 | 2.382 | 2.649 |
| 70 | .847 | 1.294 | 1.667 | 1.994 | 2.381 | 2.648 |
| 71 | .847 | 1.294 | 1.667 | 1.994 | 2.380 | 2.647 |
| 72 | .847 | 1.293 | 1.666 | 1.993 | 2.379 | 2.646 |
| 73 | .847 | 1.293 | 1.666 | 1.993 | 2.379 | 2.645 |
| 74 | .847 | 1.293 | 1.666 | 1.993 | 2.378 | 2.644 |
| 75 | .846 | 1.293 | 1.665 | 1.992 | 2.377 | 2.643 |
| 76 | .846 | 1.293 | 1.665 | 1.992 | 2.376 | 2.642 |
| 77 | .846 | 1.293 | 1.665 | 1.991 | 2.376 | 2.641 |
| 78 | .846 | 1.292 | 1.665 | 1.991 | 2.375 | 2.640 |
| 79 | .846 | 1.292 | 1.664 | 1.990 | 2.374 | 2.639 |

TABLE 3 CHI-SQUARE DISTRIBUTION



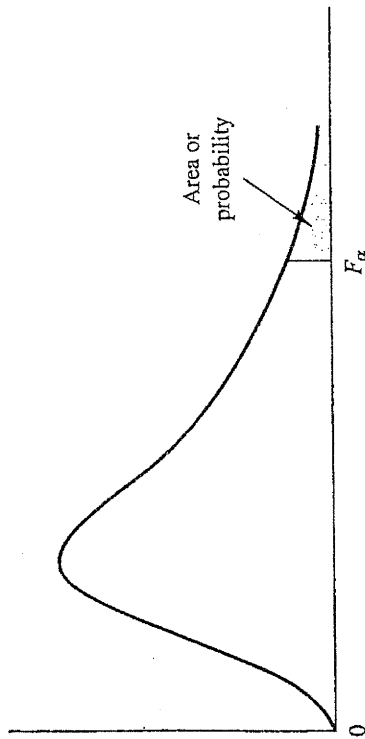
Entries in the table give χ_{α}^2 values, where α is the area or probability in the upper tail of the chi-square distribution. For example, with 10 degrees of freedom and a .01 area in the upper tail, $\chi_{.01}^2 = 23.209$.

| Degrees of Freedom | Area in Upper Tail | | | | | | | | | |
|--------------------|--------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| | .995 | .99 | .975 | .95 | .90 | .10 | .05 | .025 | .01 | .005 |
| 1 | .000 | .000 | .001 | .004 | .016 | 2.706 | 3.841 | 5.024 | 6.635 | 7.879 |
| 2 | .010 | .020 | .051 | .103 | .211 | 4.605 | 5.991 | 7.378 | 9.210 | 10.597 |
| 3 | .072 | .115 | .216 | .352 | .584 | 6.251 | 7.815 | 9.348 | 11.345 | 12.838 |
| 4 | .207 | .297 | .484 | .711 | 1.064 | 7.779 | 9.488 | 11.143 | 13.277 | 14.860 |
| 5 | .412 | .554 | .831 | 1.145 | 1.610 | 9.236 | 11.070 | 12.832 | 15.086 | 16.750 |
| 6 | .676 | .872 | 1.237 | 1.635 | 2.204 | 10.645 | 12.592 | 14.449 | 16.812 | 18.548 |
| 7 | .989 | 1.239 | 1.690 | 2.167 | 2.833 | 12.017 | 14.067 | 16.013 | 18.475 | 20.278 |
| 8 | 1.344 | 1.647 | 2.180 | 2.733 | 3.490 | 13.362 | 15.507 | 17.535 | 20.090 | 21.955 |
| 9 | 1.735 | 2.088 | 2.700 | 3.325 | 4.168 | 14.684 | 16.919 | 19.023 | 21.666 | 23.589 |
| 10 | 2.156 | 2.558 | 3.247 | 3.940 | 4.865 | 15.987 | 18.307 | 20.483 | 23.209 | 25.188 |
| 11 | 2.603 | 3.053 | 3.816 | 4.575 | 5.578 | 17.275 | 19.675 | 21.920 | 24.725 | 26.757 |
| 12 | 3.074 | 3.571 | 4.404 | 5.226 | 6.304 | 18.549 | 21.026 | 23.337 | 26.217 | 28.300 |
| 13 | 3.565 | 4.107 | 5.009 | 5.892 | 7.041 | 19.812 | 22.362 | 24.736 | 27.688 | 29.819 |
| 14 | 4.075 | 4.660 | 5.629 | 6.571 | 7.790 | 21.064 | 23.685 | 26.119 | 29.141 | 31.319 |
| 15 | 4.601 | 5.229 | 6.262 | 7.261 | 8.547 | 22.307 | 24.996 | 27.488 | 30.578 | 32.801 |
| 16 | 5.142 | 5.812 | 6.908 | 7.962 | 9.312 | 23.542 | 26.296 | 28.845 | 32.000 | 34.267 |
| 17 | 5.697 | 6.408 | 7.564 | 8.672 | 10.085 | 24.769 | 27.587 | 30.191 | 33.409 | 35.718 |
| 18 | 6.265 | 7.015 | 8.231 | 9.390 | 10.865 | 25.989 | 28.869 | 31.526 | 34.805 | 37.156 |
| 19 | 6.844 | 7.633 | 8.907 | 10.117 | 11.651 | 27.204 | 30.144 | 32.852 | 36.191 | 38.582 |
| 20 | 7.434 | 8.260 | 9.591 | 10.851 | 12.443 | 28.412 | 31.410 | 34.170 | 37.566 | 39.997 |
| 21 | 8.034 | 8.897 | 10.283 | 11.591 | 13.240 | 29.615 | 32.671 | 35.479 | 38.932 | 41.401 |
| 22 | 8.643 | 9.542 | 10.982 | 12.338 | 14.041 | 30.813 | 33.924 | 36.781 | 40.289 | 42.796 |
| 23 | 9.260 | 10.196 | 11.689 | 13.091 | 14.848 | 32.007 | 35.172 | 38.076 | 41.638 | 44.181 |
| 24 | 9.886 | 10.856 | 12.401 | 13.848 | 15.659 | 33.196 | 36.415 | 39.364 | 42.980 | 45.558 |
| 25 | 10.520 | 11.524 | 13.120 | 14.611 | 16.473 | 34.382 | 37.652 | 40.646 | 44.314 | 46.928 |
| 26 | 11.160 | 12.198 | 13.844 | 15.379 | 17.292 | 35.563 | 38.885 | 41.923 | 45.642 | 48.290 |
| 27 | 11.808 | 12.878 | 14.573 | 16.151 | 18.114 | 36.741 | 40.113 | 43.195 | 46.963 | 49.645 |
| 28 | 12.461 | 13.565 | 15.308 | 16.928 | 18.939 | 37.916 | 41.337 | 44.461 | 48.278 | 50.994 |
| 29 | 13.121 | 14.256 | 16.047 | 17.708 | 19.768 | 39.087 | 42.557 | 45.722 | 49.588 | 52.335 |

TABLE 3 CHI-SQUARE DISTRIBUTION (*Continued*)

| Degrees of Freedom | Area in Upper Tail | | | | | | | | | |
|-----------------------|--------------------|--------|--------|--------|--------|---------|---------|---------|---------|---------|
| | .995 | .99 | .975 | .95 | .90 | .10 | .05 | .025 | .01 | .005 |
| 30 | 13.787 | 14.953 | 16.791 | 18.493 | 20.599 | 40.256 | 43.773 | 46.979 | 50.892 | 53.672 |
| 35 | 17.192 | 18.509 | 20.569 | 22.465 | 24.797 | 46.059 | 49.802 | 53.203 | 57.342 | 60.275 |
| 40 | 20.707 | 22.164 | 24.433 | 26.509 | 29.051 | 51.805 | 55.758 | 59.342 | 63.691 | 66.766 |
| 45 | 24.311 | 25.901 | 28.366 | 30.612 | 33.350 | 57.505 | 61.656 | 65.410 | 69.957 | 73.166 |
| 50 | 27.991 | 29.707 | 32.357 | 34.764 | 37.689 | 63.167 | 67.505 | 71.420 | 76.154 | 79.490 |
| 55 | 31.735 | 33.571 | 36.398 | 38.958 | 42.060 | 68.796 | 73.311 | 77.380 | 82.292 | 85.749 |
| 60 | 35.534 | 37.485 | 40.482 | 43.188 | 46.459 | 74.397 | 79.082 | 83.298 | 88.379 | 91.952 |
| 65 | 39.383 | 41.444 | 44.603 | 47.450 | 50.883 | 79.973 | 84.821 | 89.177 | 94.422 | 98.105 |
| 70 | 43.275 | 45.442 | 48.758 | 51.739 | 55.329 | 85.527 | 90.531 | 95.023 | 100.425 | 104.215 |
| 75 | 47.206 | 49.475 | 52.942 | 56.054 | 59.795 | 91.061 | 96.217 | 100.839 | 106.393 | 110.285 |
| 80 | 51.172 | 53.540 | 57.153 | 60.391 | 64.278 | 96.578 | 101.879 | 106.629 | 112.329 | 116.321 |
| 85 | 55.170 | 57.634 | 61.389 | 64.749 | 68.777 | 102.079 | 107.522 | 112.393 | 118.236 | 122.324 |
| 90 | 59.196 | 61.754 | 65.647 | 69.126 | 73.291 | 107.565 | 113.145 | 118.136 | 124.116 | 128.299 |
| 95 | 63.250 | 65.898 | 69.925 | 73.520 | 77.818 | 113.038 | 118.752 | 123.858 | 129.973 | 134.247 |
| 100 | 67.328 | 70.065 | 74.222 | 77.929 | 82.358 | 118.498 | 124.342 | 129.561 | 135.807 | 140.170 |

TABLE 4 F DISTRIBUTION



Entries in the table give F_{α} values, where α is the area or probability in the upper tail of the F distribution. For example, with 4 numerator degrees of freedom, 8 denominator degrees of freedom, and a .05 area in the upper tail, $F_{.05} = 3.84$.

| Denominator Degrees of Freedom | Area in Upper Tail | Numerator Degrees of Freedom | | | | | | | | | | | | | | | | | |
|--------------------------------------|--------------------------|------------------------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 15 | 20 | 25 | 30 | 40 | 60 | 100 | 1000 |
| 1 | .10 | 39.86 | 49.50 | 53.59 | 55.83 | 57.24 | 58.20 | 58.91 | 59.44 | 59.86 | 60.19 | 61.22 | 61.74 | 62.05 | 62.26 | 62.33 | 62.79 | 63.01 | 63.30 |
| | .05 | 161.45 | 199.50 | 215.71 | 224.58 | 230.16 | 233.99 | 236.77 | 238.98 | 240.54 | 241.88 | 245.95 | 248.02 | 249.26 | 250.10 | 251.14 | 252.20 | 253.04 | 254.19 |
| | .025 | 647.79 | 799.48 | 864.15 | 899.60 | 921.83 | 937.11 | 948.20 | 956.64 | 963.28 | 968.63 | 984.87 | 993.08 | 998.09 | 1001.40 | 1005.60 | 1009.79 | 1013.16 | 1017.76 |
| | .01 | 4052.18 | 4999.34 | 5403.53 | 5624.26 | 5763.96 | 5858.95 | 5928.33 | 5980.95 | 6022.40 | 6055.93 | 6156.97 | 6208.66 | 6239.86 | 6260.35 | 6286.43 | 6312.97 | 6333.92 | 6362.80 |
| 2 | .10 | 8.53 | 9.00 | 9.16 | 9.24 | 9.29 | 9.33 | 9.35 | 9.37 | 9.38 | 9.39 | 9.42 | 9.44 | 9.45 | 9.46 | 9.47 | 9.47 | 9.48 | 9.49 |
| | .05 | 18.51 | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.35 | 19.37 | 19.38 | 19.40 | 19.43 | 19.45 | 19.46 | 19.46 | 19.47 | 19.48 | 19.49 | 19.49 |
| | .025 | 38.51 | 39.00 | 39.17 | 39.25 | 39.30 | 39.33 | 39.36 | 39.37 | 39.39 | 39.40 | 39.43 | 39.45 | 39.46 | 39.46 | 39.47 | 39.48 | 39.49 | 39.50 |
| | .01 | 98.50 | 99.00 | 99.16 | 99.25 | 99.30 | 99.33 | 99.36 | 99.38 | 99.39 | 99.40 | 99.43 | 99.45 | 99.46 | 99.46 | 99.47 | 99.48 | 99.49 | 99.50 |
| 3 | .10 | 5.54 | 5.46 | 5.39 | 5.34 | 5.31 | 5.28 | 5.27 | 5.25 | 5.24 | 5.23 | 5.20 | 5.18 | 5.17 | 5.17 | 5.16 | 5.15 | 5.14 | 5.13 |
| | .05 | 10.13 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 | 8.79 | 8.70 | 8.66 | 8.63 | 8.62 | 8.59 | 8.57 | 8.55 | 8.53 |
| | .025 | 17.44 | 16.04 | 15.44 | 15.10 | 14.88 | 14.73 | 14.62 | 14.54 | 14.47 | 14.42 | 14.25 | 14.17 | 14.12 | 14.08 | 14.04 | 13.99 | 13.96 | 13.91 |
| | .01 | 34.12 | 30.82 | 29.46 | 28.71 | 28.24 | 27.91 | 27.67 | 27.49 | 27.34 | 27.23 | 26.87 | 26.69 | 26.58 | 26.50 | 26.41 | 26.32 | 26.24 | 26.14 |
| 4 | .10 | 4.54 | 4.32 | 4.19 | 4.11 | 4.05 | 4.01 | 3.98 | 3.95 | 3.94 | 3.92 | 3.87 | 3.84 | 3.83 | 3.82 | 3.80 | 3.79 | 3.78 | 3.76 |
| | .05 | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 | 5.96 | 5.86 | 5.80 | 5.77 | 5.75 | 5.72 | 5.69 | 5.66 | 5.63 |
| | .025 | 12.22 | 10.65 | 9.98 | 9.60 | 9.36 | 9.20 | 9.07 | 8.98 | 8.90 | 8.84 | 8.66 | 8.56 | 8.50 | 8.46 | 8.41 | 8.36 | 8.32 | 8.26 |
| | .01 | 21.20 | 18.00 | 16.69 | 15.98 | 15.52 | 15.21 | 14.98 | 14.80 | 14.66 | 14.55 | 14.20 | 14.02 | 13.91 | 13.84 | 13.75 | 13.65 | 13.58 | 13.47 |
| 5 | .10 | 4.06 | 3.78 | 3.62 | 3.52 | 3.45 | 3.40 | 3.37 | 3.34 | 3.32 | 3.30 | 3.24 | 3.21 | 3.19 | 3.17 | 3.16 | 3.14 | 3.13 | 3.11 |
| | .05 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 | 4.74 | 4.62 | 4.56 | 4.52 | 4.50 | 4.46 | 4.43 | 4.41 | 4.37 |
| | .025 | 10.01 | 8.43 | 7.76 | 7.39 | 7.15 | 6.98 | 6.85 | 6.76 | 6.68 | 6.62 | 6.43 | 6.33 | 6.27 | 6.23 | 6.18 | 6.12 | 6.08 | 6.02 |
| | .01 | 16.26 | 13.27 | 12.06 | 11.39 | 10.97 | 10.67 | 10.46 | 10.29 | 10.16 | 10.05 | 9.72 | 9.55 | 9.45 | 9.38 | 9.29 | 9.20 | 9.13 | 9.03 |

| Denominator Degrees of Freedom | Area in Upper Tail | Numerator Degrees of Freedom | | | | | | | | | | | | | | | | | |
|--------------------------------------|--------------------------|------------------------------|-------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 15 | 20 | 25 | 30 | 40 | 60 | 100 | 1000 |
| 6 | .10 | 3.78 | 3.46 | 3.29 | 3.18 | 3.11 | 3.05 | 3.01 | 2.98 | 2.96 | 2.94 | 2.87 | 2.84 | 2.81 | 2.80 | 2.78 | 2.76 | 2.75 | 2.72 |
| | .05 | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 | 4.06 | 3.94 | 3.87 | 3.83 | 3.81 | 3.77 | 3.74 | 3.71 | 3.67 |
| | .025 | 8.81 | 7.26 | 6.60 | 6.23 | 5.99 | 5.82 | 5.70 | 5.60 | 5.52 | 5.46 | 5.27 | 5.17 | 5.11 | 5.07 | 5.01 | 4.96 | 4.92 | 4.86 |
| | .01 | 13.75 | 10.92 | 9.78 | 9.15 | 8.75 | 8.47 | 8.26 | 8.10 | 7.98 | 7.87 | 7.56 | 7.40 | 7.30 | 7.23 | 7.14 | 7.06 | 6.99 | 6.89 |
| 7 | .10 | 3.59 | 3.26 | 3.07 | 2.96 | 2.88 | 2.83 | 2.78 | 2.75 | 2.72 | 2.70 | 2.63 | 2.59 | 2.57 | 2.56 | 2.54 | 2.51 | 2.50 | 2.47 |
| | .05 | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 | 3.64 | 3.51 | 3.44 | 3.40 | 3.38 | 3.34 | 3.30 | 3.27 | 3.23 |
| | .025 | 8.07 | 6.54 | 5.89 | 5.52 | 5.29 | 5.12 | 4.99 | 4.90 | 4.82 | 4.76 | 4.57 | 4.47 | 4.40 | 4.36 | 4.31 | 4.25 | 4.21 | 4.15 |
| | .01 | 12.25 | 9.55 | 8.45 | 7.85 | 7.46 | 7.19 | 6.99 | 6.84 | 6.72 | 6.62 | 6.31 | 6.16 | 6.06 | 5.99 | 5.91 | 5.82 | 5.75 | 5.66 |
| 8 | .10 | 3.46 | 3.11 | 2.92 | 2.81 | 2.73 | 2.67 | 2.62 | 2.59 | 2.56 | 2.54 | 2.46 | 2.42 | 2.40 | 2.38 | 2.36 | 2.34 | 2.32 | 2.30 |
| | .05 | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 | 3.35 | 3.22 | 3.15 | 3.11 | 3.08 | 3.04 | 3.01 | 2.97 | 2.93 |
| | .025 | 7.57 | 6.06 | 5.42 | 5.05 | 4.82 | 4.65 | 4.53 | 4.43 | 4.36 | 4.30 | 4.10 | 4.00 | 3.94 | 3.89 | 3.84 | 3.78 | 3.74 | 3.68 |
| | .01 | 11.26 | 8.65 | 7.59 | 7.01 | 6.63 | 6.37 | 6.18 | 6.03 | 5.91 | 5.81 | 5.52 | 5.36 | 5.26 | 5.20 | 5.12 | 5.03 | 4.96 | 4.87 |
| 9 | .10 | 3.36 | 3.01 | 2.81 | 2.69 | 2.61 | 2.55 | 2.51 | 2.47 | 2.44 | 2.42 | 2.34 | 2.30 | 2.27 | 2.25 | 2.23 | 2.21 | 2.19 | 2.16 |
| | .05 | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 | 3.14 | 3.01 | 2.94 | 2.89 | 2.86 | 2.83 | 2.79 | 2.76 | 2.71 |
| | .025 | 7.21 | 5.71 | 5.08 | 4.72 | 4.48 | 4.32 | 4.20 | 4.10 | 4.03 | 3.96 | 3.77 | 3.67 | 3.60 | 3.56 | 3.51 | 3.45 | 3.40 | 3.34 |
| | .01 | 10.56 | 8.02 | 6.99 | 6.42 | 6.06 | 5.80 | 5.61 | 5.47 | 5.35 | 5.26 | 4.96 | 4.81 | 4.71 | 4.65 | 4.57 | 4.48 | 4.41 | 4.32 |
| 10 | .10 | 3.29 | 2.92 | 2.73 | 2.61 | 2.52 | 2.46 | 2.41 | 2.38 | 2.35 | 2.32 | 2.24 | 2.20 | 2.17 | 2.16 | 2.13 | 2.11 | 2.09 | 2.06 |
| | .05 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 | 2.98 | 2.85 | 2.77 | 2.73 | 2.70 | 2.66 | 2.62 | 2.59 | 2.54 |
| | .025 | 6.94 | 5.46 | 4.83 | 4.47 | 4.24 | 4.07 | 3.95 | 3.85 | 3.78 | 3.72 | 3.52 | 3.42 | 3.35 | 3.31 | 3.26 | 3.20 | 3.15 | 3.09 |
| | .01 | 10.04 | 7.56 | 6.55 | 5.99 | 5.64 | 5.39 | 5.20 | 5.06 | 4.94 | 4.85 | 4.56 | 4.41 | 4.31 | 4.25 | 4.17 | 4.08 | 4.01 | 3.92 |
| 11 | .10 | 3.23 | 2.86 | 2.66 | 2.54 | 2.45 | 2.39 | 2.34 | 2.30 | 2.27 | 2.25 | 2.17 | 2.12 | 2.10 | 2.08 | 2.05 | 2.03 | 2.01 | 1.98 |
| | .05 | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.90 | 2.85 | 2.72 | 2.65 | 2.60 | 2.57 | 2.53 | 2.49 | 2.46 | 2.41 |
| | .025 | 6.72 | 5.26 | 4.63 | 4.28 | 4.04 | 3.88 | 3.76 | 3.66 | 3.59 | 3.53 | 3.33 | 3.23 | 3.16 | 3.12 | 3.06 | 3.00 | 2.96 | 2.89 |
| | .01 | 9.65 | 7.21 | 6.22 | 5.67 | 5.32 | 5.07 | 4.89 | 4.74 | 4.63 | 4.54 | 4.25 | 4.10 | 4.01 | 3.94 | 3.86 | 3.78 | 3.71 | 3.61 |
| 12 | .10 | 3.18 | 2.81 | 2.61 | 2.48 | 2.39 | 2.33 | 2.28 | 2.24 | 2.21 | 2.19 | 2.10 | 2.06 | 2.03 | 2.01 | 1.99 | 1.96 | 1.94 | 1.91 |
| | .05 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 | 2.75 | 2.62 | 2.54 | 2.50 | 2.47 | 2.43 | 2.38 | 2.35 | 2.30 |
| | .025 | 6.55 | 5.10 | 4.47 | 4.12 | 3.89 | 3.73 | 3.61 | 3.51 | 3.44 | 3.37 | 3.18 | 3.07 | 3.01 | 2.96 | 2.91 | 2.85 | 2.80 | 2.73 |
| | .01 | 9.33 | 6.93 | 5.95 | 5.41 | 5.06 | 4.82 | 4.64 | 4.50 | 4.39 | 4.30 | 4.01 | 3.86 | 3.76 | 3.70 | 3.62 | 3.54 | 3.47 | 3.37 |
| 13 | .10 | 3.14 | 2.76 | 2.56 | 2.43 | 2.35 | 2.28 | 2.23 | 2.20 | 2.16 | 2.14 | 2.05 | 2.01 | 1.98 | 1.96 | 1.93 | 1.90 | 1.88 | 1.85 |
| | .05 | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.71 | 2.67 | 2.53 | 2.46 | 2.41 | 2.38 | 2.34 | 2.30 | 2.26 | 2.21 |
| | .025 | 6.41 | 4.97 | 4.35 | 4.00 | 3.77 | 3.60 | 3.48 | 3.39 | 3.31 | 3.25 | 3.05 | 2.95 | 2.88 | 2.84 | 2.78 | 2.72 | 2.67 | 2.60 |
| | .01 | 9.07 | 6.70 | 5.74 | 5.21 | 4.86 | 4.62 | 4.44 | 4.30 | 4.19 | 4.10 | 3.82 | 3.66 | 3.57 | 3.51 | 3.43 | 3.34 | 3.27 | 3.18 |
| 14 | .10 | 3.10 | 2.73 | 2.52 | 2.39 | 2.31 | 2.24 | 2.19 | 2.15 | 2.12 | 2.10 | 2.01 | 1.96 | 1.93 | 1.91 | 1.89 | 1.86 | 1.83 | 1.80 |
| | .05 | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.65 | 2.60 | 2.46 | 2.39 | 2.34 | 2.31 | 2.27 | 2.22 | 2.19 | 2.14 |
| | .025 | 6.30 | 4.86 | 4.24 | 3.89 | 3.66 | 3.50 | 3.38 | 3.29 | 3.21 | 3.15 | 2.95 | 2.84 | 2.78 | 2.73 | 2.67 | 2.61 | 2.56 | 2.50 |
| | .01 | 8.86 | 6.51 | 5.56 | 5.04 | 4.69 | 4.46 | 4.28 | 4.14 | 4.03 | 3.94 | 3.66 | 3.51 | 3.41 | 3.35 | 3.27 | 3.18 | 3.11 | 3.02 |
| 15 | .10 | 3.07 | 2.70 | 2.49 | 2.36 | 2.27 | 2.21 | 2.16 | 2.12 | 2.09 | 2.06 | 1.97 | 1.92 | 1.89 | 1.87 | 1.85 | 1.82 | 1.79 | 1.76 |
| | .05 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 | 2.54 | 2.40 | 2.33 | 2.28 | 2.25 | 2.20 | 2.16 | 2.12 | 2.07 |
| | .025 | 6.20 | 4.77 | 4.15 | 3.80 | 3.58 | 3.41 | 3.29 | 3.20 | 3.12 | 3.06 | 2.86 | 2.76 | 2.69 | 2.64 | 2.59 | 2.52 | 2.47 | 2.40 |
| | .01 | 8.68 | 6.36 | 5.42 | 4.89 | 4.56 | 4.32 | 4.14 | 4.00 | 3.89 | 3.80 | 3.52 | 3.37 | 3.28 | 3.21 | 3.13 | 3.05 | 2.98 | 2.88 |

TABLE 4 F DISTRIBUTION (continued)

| Denominator Degrees of Freedom | Area in Upper Tail | Numerator Degrees of Freedom | | | | | | | | | | | | | | | | | |
|--------------------------------------|--------------------------|------------------------------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 15 | 20 | 25 | 30 | 40 | 60 | 100 | 1000 |
| 16 | .10 | 3.05 | 2.67 | 2.46 | 2.33 | 2.24 | 2.18 | 2.13 | 2.09 | 2.06 | 2.03 | 1.94 | 1.89 | 1.86 | 1.84 | 1.81 | 1.78 | 1.76 | 1.72 |
| | .05 | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.54 | 2.49 | 2.35 | 2.28 | 2.23 | 2.19 | 2.15 | 2.11 | 2.07 | 2.02 |
| | .025 | 6.12 | 4.69 | 4.08 | 3.73 | 3.50 | 3.34 | 3.22 | 3.12 | 3.05 | 2.99 | 2.79 | 2.68 | 2.61 | 2.57 | 2.51 | 2.45 | 2.40 | 2.32 |
| | .01 | 8.53 | 6.23 | 5.29 | 4.77 | 4.44 | 4.20 | 4.03 | 3.89 | 3.78 | 3.69 | 3.41 | 3.26 | 3.16 | 3.10 | 3.02 | 2.93 | 2.86 | 2.76 |
| 17 | .10 | 3.03 | 2.64 | 2.44 | 2.31 | 2.22 | 2.15 | 2.10 | 2.06 | 2.03 | 2.00 | 1.91 | 1.86 | 1.83 | 1.81 | 1.78 | 1.75 | 1.73 | 1.69 |
| | .05 | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.49 | 2.45 | 2.31 | 2.23 | 2.18 | 2.15 | 2.10 | 2.06 | 2.02 | 1.97 |
| | .025 | 6.04 | 4.62 | 4.01 | 3.66 | 3.44 | 3.28 | 3.16 | 3.06 | 2.98 | 2.92 | 2.72 | 2.62 | 2.55 | 2.50 | 2.44 | 2.38 | 2.33 | 2.26 |
| | .01 | 8.40 | 6.11 | 5.19 | 4.67 | 4.34 | 4.10 | 3.93 | 3.79 | 3.68 | 3.59 | 3.31 | 3.16 | 3.07 | 3.00 | 2.92 | 2.83 | 2.76 | 2.66 |
| 18 | .10 | 3.01 | 2.62 | 2.42 | 2.29 | 2.20 | 2.13 | 2.08 | 2.04 | 2.00 | 1.98 | 1.89 | 1.84 | 1.80 | 1.78 | 1.75 | 1.72 | 1.70 | 1.66 |
| | .05 | 4.41 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.46 | 2.41 | 2.27 | 2.19 | 2.14 | 2.11 | 2.06 | 2.02 | 1.98 | 1.92 |
| | .025 | 5.98 | 4.56 | 3.95 | 3.61 | 3.38 | 3.22 | 3.10 | 3.01 | 2.93 | 2.87 | 2.67 | 2.56 | 2.49 | 2.44 | 2.38 | 2.32 | 2.27 | 2.20 |
| | .01 | 8.29 | 6.01 | 5.09 | 4.58 | 4.25 | 4.01 | 3.84 | 3.71 | 3.60 | 3.51 | 3.23 | 3.08 | 2.98 | 2.92 | 2.84 | 2.75 | 2.68 | 2.58 |
| 19 | .10 | 2.99 | 2.61 | 2.40 | 2.27 | 2.18 | 2.11 | 2.06 | 2.02 | 1.98 | 1.96 | 1.86 | 1.81 | 1.78 | 1.76 | 1.73 | 1.70 | 1.67 | 1.64 |
| | .05 | 4.38 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.42 | 2.38 | 2.23 | 2.16 | 2.11 | 2.07 | 2.03 | 1.98 | 1.94 | 1.88 |
| | .025 | 5.92 | 4.51 | 3.90 | 3.56 | 3.33 | 3.17 | 3.05 | 2.96 | 2.88 | 2.82 | 2.62 | 2.51 | 2.44 | 2.39 | 2.33 | 2.27 | 2.22 | 2.14 |
| | .01 | 8.18 | 5.93 | 5.01 | 4.50 | 4.17 | 3.94 | 3.77 | 3.63 | 3.52 | 3.43 | 3.15 | 3.00 | 2.91 | 2.84 | 2.76 | 2.67 | 2.60 | 2.50 |
| 20 | .10 | 2.97 | 2.59 | 2.38 | 2.25 | 2.16 | 2.09 | 2.04 | 2.00 | 1.96 | 1.94 | 1.84 | 1.79 | 1.76 | 1.74 | 1.71 | 1.68 | 1.65 | 1.61 |
| | .05 | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 | 2.35 | 2.20 | 2.12 | 2.07 | 2.04 | 1.99 | 1.95 | 1.91 | 1.85 |
| | .025 | 5.87 | 4.46 | 3.86 | 3.51 | 3.29 | 3.13 | 3.01 | 2.91 | 2.84 | 2.77 | 2.57 | 2.46 | 2.40 | 2.35 | 2.29 | 2.22 | 2.17 | 2.09 |
| | .01 | 8.10 | 5.85 | 4.94 | 4.43 | 4.10 | 3.87 | 3.70 | 3.56 | 3.46 | 3.37 | 3.09 | 2.94 | 2.84 | 2.78 | 2.69 | 2.61 | 2.54 | 2.43 |
| 21 | .10 | 2.96 | 2.57 | 2.36 | 2.23 | 2.14 | 2.08 | 2.02 | 1.98 | 1.95 | 1.92 | 1.83 | 1.78 | 1.74 | 1.72 | 1.69 | 1.66 | 1.63 | 1.59 |
| | .05 | 4.32 | 3.47 | 3.07 | 2.84 | 2.68 | 2.57 | 2.49 | 2.42 | 2.37 | 2.32 | 2.18 | 2.10 | 2.05 | 2.01 | 1.96 | 1.92 | 1.88 | 1.82 |
| | .025 | 5.83 | 4.42 | 3.82 | 3.48 | 3.25 | 3.09 | 2.97 | 2.87 | 2.80 | 2.73 | 2.53 | 2.42 | 2.36 | 2.31 | 2.25 | 2.18 | 2.13 | 2.05 |
| | .01 | 8.02 | 5.78 | 4.87 | 4.37 | 4.04 | 3.81 | 3.64 | 3.51 | 3.40 | 3.31 | 3.03 | 2.88 | 2.79 | 2.72 | 2.64 | 2.55 | 2.48 | 2.37 |
| 22 | .10 | 2.95 | 2.56 | 2.35 | 2.22 | 2.13 | 2.06 | 2.01 | 1.97 | 1.93 | 1.90 | 1.81 | 1.76 | 1.73 | 1.70 | 1.67 | 1.64 | 1.61 | 1.57 |
| | .05 | 4.30 | 3.44 | 3.05 | 2.82 | 2.66 | 2.55 | 2.46 | 2.40 | 2.34 | 2.30 | 2.15 | 2.07 | 2.02 | 1.98 | 1.94 | 1.89 | 1.85 | 1.79 |
| | .025 | 5.79 | 4.38 | 3.78 | 3.44 | 3.22 | 3.05 | 2.93 | 2.84 | 2.76 | 2.70 | 2.50 | 2.39 | 2.32 | 2.27 | 2.21 | 2.14 | 2.09 | 2.01 |
| | .01 | 7.95 | 5.72 | 4.82 | 4.31 | 3.99 | 3.76 | 3.59 | 3.45 | 3.35 | 3.26 | 2.98 | 2.83 | 2.73 | 2.67 | 2.58 | 2.50 | 2.42 | 2.32 |
| 23 | .10 | 2.94 | 2.55 | 2.34 | 2.21 | 2.11 | 2.05 | 1.99 | 1.95 | 1.92 | 1.89 | 1.80 | 1.74 | 1.71 | 1.69 | 1.66 | 1.62 | 1.59 | 1.55 |
| | .05 | 4.28 | 3.42 | 3.03 | 2.80 | 2.64 | 2.53 | 2.44 | 2.37 | 2.32 | 2.27 | 2.13 | 2.05 | 2.00 | 1.96 | 1.91 | 1.86 | 1.82 | 1.76 |
| | .025 | 5.75 | 4.35 | 3.75 | 3.41 | 3.18 | 3.02 | 2.90 | 2.81 | 2.73 | 2.67 | 2.47 | 2.36 | 2.29 | 2.24 | 2.18 | 2.11 | 2.06 | 1.98 |
| | .01 | 7.88 | 5.66 | 4.76 | 4.26 | 3.94 | 3.71 | 3.54 | 3.41 | 3.30 | 3.21 | 2.93 | 2.78 | 2.69 | 2.62 | 2.54 | 2.45 | 2.37 | 2.27 |
| 24 | .10 | 2.93 | 2.54 | 2.33 | 2.19 | 2.10 | 2.04 | 1.98 | 1.94 | 1.91 | 1.88 | 1.78 | 1.73 | 1.70 | 1.67 | 1.64 | 1.61 | 1.58 | 1.54 |
| | .05 | 4.26 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 | 2.25 | 2.11 | 2.03 | 1.97 | 1.94 | 1.89 | 1.84 | 1.80 | 1.74 |
| | .025 | 5.72 | 4.32 | 3.72 | 3.38 | 3.15 | 2.99 | 2.87 | 2.78 | 2.70 | 2.64 | 2.44 | 2.33 | 2.26 | 2.21 | 2.15 | 2.08 | 2.02 | 1.94 |
| | .01 | 7.82 | 5.61 | 4.72 | 4.22 | 3.90 | 3.67 | 3.50 | 3.36 | 3.26 | 3.17 | 2.89 | 2.74 | 2.64 | 2.58 | 2.49 | 2.40 | 2.33 | 2.22 |

| Denominator Degrees of Freedom | Area in Upper Tail | Numerator Degrees of Freedom | | | | | | | | | | | | | | | | | |
|--------------------------------------|--------------------------|------------------------------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 15 | 20 | 25 | 30 | 40 | 60 | 100 | 1000 |
| 25 | .10 | 2.92 | 2.53 | 2.32 | 2.18 | 2.09 | 2.02 | 1.97 | 1.93 | 1.89 | 1.87 | 1.77 | 1.72 | 1.68 | 1.66 | 1.63 | 1.59 | 1.56 | 1.52 |
| | .05 | 4.24 | 3.39 | 2.99 | 2.76 | 2.60 | 2.49 | 2.40 | 2.34 | 2.28 | 2.24 | 2.09 | 2.01 | 1.96 | 1.92 | 1.87 | 1.82 | 1.78 | 1.72 |
| | .025 | 5.69 | 4.29 | 3.69 | 3.35 | 3.13 | 2.97 | 2.85 | 2.75 | 2.68 | 2.61 | 2.41 | 2.30 | 2.23 | 2.18 | 2.12 | 2.05 | 2.00 | 1.91 |
| | .01 | 7.77 | 5.57 | 4.68 | 4.18 | 3.85 | 3.63 | 3.46 | 3.32 | 3.22 | 3.13 | 2.85 | 2.70 | 2.60 | 2.54 | 2.45 | 2.36 | 2.29 | 2.18 |
| | .10 | 2.91 | 2.52 | 2.31 | 2.17 | 2.08 | 2.01 | 1.96 | 1.92 | 1.88 | 1.86 | 1.76 | 1.71 | 1.67 | 1.65 | 1.61 | 1.58 | 1.55 | 1.51 |
| 26 | .05 | 4.23 | 3.37 | 2.98 | 2.74 | 2.59 | 2.47 | 2.39 | 2.32 | 2.27 | 2.22 | 2.07 | 1.99 | 1.94 | 1.90 | 1.85 | 1.80 | 1.76 | 1.70 |
| | .025 | 5.66 | 4.27 | 3.67 | 3.33 | 3.10 | 2.94 | 2.82 | 2.73 | 2.65 | 2.59 | 2.39 | 2.28 | 2.21 | 2.16 | 2.09 | 2.03 | 1.97 | 1.89 |
| | .01 | 7.72 | 5.53 | 4.64 | 4.14 | 3.82 | 3.59 | 3.42 | 3.29 | 3.18 | 3.09 | 2.81 | 2.66 | 2.57 | 2.50 | 2.42 | 2.33 | 2.25 | 2.14 |
| | .10 | 2.90 | 2.51 | 2.30 | 2.17 | 2.07 | 2.00 | 1.95 | 1.91 | 1.87 | 1.85 | 1.75 | 1.70 | 1.66 | 1.64 | 1.60 | 1.57 | 1.54 | 1.50 |
| | .05 | 4.21 | 3.35 | 2.96 | 2.73 | 2.57 | 2.46 | 2.37 | 2.31 | 2.25 | 2.20 | 2.06 | 1.97 | 1.92 | 1.88 | 1.84 | 1.79 | 1.74 | 1.68 |
| 27 | .025 | 5.63 | 4.24 | 3.65 | 3.31 | 3.08 | 2.92 | 2.80 | 2.71 | 2.63 | 2.57 | 2.36 | 2.25 | 2.18 | 2.13 | 2.07 | 2.00 | 1.94 | 1.86 |
| | .01 | 7.68 | 5.49 | 4.60 | 4.11 | 3.78 | 3.56 | 3.39 | 3.26 | 3.15 | 3.06 | 2.78 | 2.63 | 2.54 | 2.47 | 2.38 | 2.29 | 2.22 | 2.11 |
| | .10 | 2.89 | 2.50 | 2.29 | 2.16 | 2.06 | 2.00 | 1.94 | 1.90 | 1.87 | 1.84 | 1.74 | 1.69 | 1.65 | 1.63 | 1.59 | 1.56 | 1.53 | 1.48 |
| | .05 | 4.20 | 3.34 | 2.95 | 2.71 | 2.56 | 2.45 | 2.36 | 2.29 | 2.24 | 2.19 | 2.04 | 1.96 | 1.91 | 1.87 | 1.82 | 1.77 | 1.73 | 1.66 |
| | .025 | 5.61 | 4.22 | 3.63 | 3.29 | 3.06 | 2.90 | 2.78 | 2.69 | 2.61 | 2.55 | 2.34 | 2.23 | 2.16 | 2.11 | 2.05 | 1.98 | 1.92 | 1.84 |
| 28 | .01 | 7.64 | 5.45 | 4.57 | 4.07 | 3.75 | 3.53 | 3.36 | 3.23 | 3.12 | 3.03 | 2.75 | 2.60 | 2.51 | 2.44 | 2.35 | 2.26 | 2.19 | 2.08 |
| | .10 | 2.89 | 2.50 | 2.28 | 2.15 | 2.06 | 2.00 | 1.93 | 1.89 | 1.86 | 1.83 | 1.73 | 1.68 | 1.64 | 1.62 | 1.58 | 1.55 | 1.52 | 1.47 |
| | .05 | 4.18 | 3.33 | 2.93 | 2.70 | 2.55 | 2.43 | 2.35 | 2.28 | 2.22 | 2.18 | 2.03 | 1.94 | 1.89 | 1.85 | 1.81 | 1.75 | 1.71 | 1.65 |
| | .025 | 5.59 | 4.20 | 3.61 | 3.27 | 3.04 | 2.88 | 2.76 | 2.67 | 2.59 | 2.53 | 2.32 | 2.21 | 2.14 | 2.09 | 2.03 | 1.96 | 1.90 | 1.82 |
| | .01 | 7.60 | 5.42 | 4.54 | 4.04 | 3.73 | 3.50 | 3.33 | 3.20 | 3.09 | 3.00 | 2.73 | 2.57 | 2.48 | 2.41 | 2.33 | 2.23 | 2.16 | 2.05 |
| 29 | .10 | 2.88 | 2.49 | 2.28 | 2.14 | 2.05 | 1.98 | 1.93 | 1.88 | 1.85 | 1.82 | 1.72 | 1.67 | 1.63 | 1.61 | 1.57 | 1.54 | 1.51 | 1.46 |
| | .05 | 4.17 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 | 2.27 | 2.21 | 2.16 | 2.01 | 1.93 | 1.88 | 1.84 | 1.79 | 1.74 | 1.70 | 1.63 |
| | .025 | 5.57 | 4.18 | 3.59 | 3.25 | 3.03 | 2.87 | 2.75 | 2.65 | 2.57 | 2.51 | 2.31 | 2.20 | 2.12 | 2.07 | 2.01 | 1.94 | 1.88 | 1.80 |
| | .01 | 7.56 | 5.39 | 4.51 | 4.02 | 3.70 | 3.47 | 3.30 | 3.17 | 3.07 | 2.98 | 2.70 | 2.55 | 2.45 | 2.39 | 2.30 | 2.21 | 2.13 | 2.02 |
| | .10 | 2.84 | 2.44 | 2.23 | 2.09 | 2.00 | 1.93 | 1.87 | 1.83 | 1.79 | 1.76 | 1.66 | 1.61 | 1.57 | 1.54 | 1.51 | 1.47 | 1.43 | 1.38 |
| 40 | .05 | 4.08 | 3.23 | 2.84 | 2.61 | 2.45 | 2.34 | 2.25 | 2.18 | 2.12 | 2.08 | 1.92 | 1.84 | 1.78 | 1.74 | 1.69 | 1.64 | 1.59 | 1.52 |
| | .025 | 5.42 | 4.05 | 3.46 | 3.13 | 2.90 | 2.74 | 2.62 | 2.53 | 2.45 | 2.39 | 2.18 | 2.07 | 1.99 | 1.94 | 1.88 | 1.80 | 1.74 | 1.65 |
| | .01 | 7.31 | 5.18 | 4.31 | 3.83 | 3.51 | 3.29 | 3.12 | 2.99 | 2.89 | 2.80 | 2.52 | 2.37 | 2.27 | 2.20 | 2.11 | 2.02 | 1.94 | 1.82 |
| | .10 | 2.79 | 2.39 | 2.18 | 2.04 | 1.95 | 1.87 | 1.82 | 1.77 | 1.74 | 1.71 | 1.60 | 1.54 | 1.50 | 1.48 | 1.44 | 1.40 | 1.36 | 1.30 |
| | .05 | 4.00 | 3.15 | 2.76 | 2.53 | 2.37 | 2.25 | 2.17 | 2.10 | 2.04 | 1.99 | 1.84 | 1.75 | 1.68 | 1.65 | 1.59 | 1.53 | 1.48 | 1.40 |
| 60 | .025 | 5.29 | 3.93 | 3.34 | 3.01 | 2.79 | 2.63 | 2.51 | 2.41 | 2.33 | 2.27 | 2.06 | 1.94 | 1.87 | 1.82 | 1.74 | 1.67 | 1.60 | 1.49 |
| | .01 | 7.08 | 4.98 | 4.13 | 3.65 | 3.34 | 3.12 | 2.95 | 2.82 | 2.72 | 2.63 | 2.35 | 2.20 | 2.10 | 2.03 | 1.94 | 1.84 | 1.75 | 1.62 |
| | .10 | 2.76 | 2.36 | 2.14 | 2.00 | 1.91 | 1.83 | 1.78 | 1.73 | 1.69 | 1.66 | 1.56 | 1.49 | 1.45 | 1.42 | 1.38 | 1.34 | 1.29 | 1.22 |
| | .05 | 3.94 | 3.09 | 2.70 | 2.46 | 2.31 | 2.19 | 2.10 | 2.03 | 1.97 | 1.93 | 1.77 | 1.68 | 1.62 | 1.57 | 1.52 | 1.45 | 1.39 | 1.30 |
| | .025 | 5.18 | 3.83 | 3.25 | 2.92 | 2.70 | 2.54 | 2.42 | 2.32 | 2.24 | 2.18 | 1.97 | 1.85 | 1.77 | 1.71 | 1.64 | 1.56 | 1.48 | 1.36 |
| 100 | .01 | 6.90 | 4.82 | 3.98 | 3.51 | 3.21 | 2.99 | 2.82 | 2.69 | 2.59 | 2.50 | 2.22 | 2.07 | 1.97 | 1.89 | 1.80 | 1.69 | 1.60 | 1.45 |
| | .10 | 2.71 | 2.31 | 2.09 | 1.95 | 1.85 | 1.78 | 1.72 | 1.68 | 1.64 | 1.61 | 1.49 | 1.43 | 1.38 | 1.35 | 1.30 | 1.25 | 1.20 | 1.08 |
| | .05 | 3.85 | 3.00 | 2.61 | 2.38 | 2.22 | 2.11 | 2.02 | 1.95 | 1.89 | 1.84 | 1.68 | 1.58 | 1.52 | 1.47 | 1.41 | 1.33 | 1.26 | 1.11 |
| | .025 | 5.04 | 3.70 | 3.13 | 2.80 | 2.58 | 2.42 | 2.30 | 2.20 | 2.13 | 2.06 | 1.85 | 1.72 | 1.64 | 1.58 | 1.50 | 1.41 | 1.32 | 1.13 |
| | .01 | 6.66 | 4.63 | 3.80 | 3.34 | 3.04 | 2.82 | 2.66 | 2.53 | 2.43 | 2.34 | 2.06 | 1.90 | 1.79 | 1.72 | 1.61 | 1.50 | 1.38 | 1.16 |

TABLE 6 VALUES OF $e^{-\mu}$

| μ | $e^{-\mu}$ | μ | $e^{-\mu}$ | μ | $e^{-\mu}$ |
|-------|------------|-------|------------|-------|------------|
| .00 | 1.0000 | 2.00 | .1353 | 4.00 | .0183 |
| .05 | .9512 | 2.05 | .1287 | 4.05 | .0174 |
| .10 | .9048 | 2.10 | .1225 | 4.10 | .0166 |
| .15 | .8607 | 2.15 | .1165 | 4.15 | .0158 |
| .20 | .8187 | 2.20 | .1108 | 4.20 | .0150 |
| .25 | .7788 | 2.25 | .1054 | 4.25 | .0143 |
| .30 | .7408 | 2.30 | .1003 | 4.30 | .0136 |
| .35 | .7047 | 2.35 | .0954 | 4.35 | .0129 |
| .40 | .6703 | 2.40 | .0907 | 4.40 | .0123 |
| .45 | .6376 | 2.45 | .0863 | 4.45 | .0117 |
| .50 | .6065 | 2.50 | .0821 | 4.50 | .0111 |
| .55 | .5769 | 2.55 | .0781 | 4.55 | .0106 |
| .60 | .5488 | 2.60 | .0743 | 4.60 | .0101 |
| .65 | .5220 | 2.65 | .0707 | 4.65 | .0096 |
| .70 | .4966 | 2.70 | .0672 | 4.70 | .0091 |
| .75 | .4724 | 2.75 | .0639 | 4.75 | .0087 |
| .80 | .4493 | 2.80 | .0608 | 4.80 | .0082 |
| .85 | .4274 | 2.85 | .0578 | 4.85 | .0078 |
| .90 | .4066 | 2.90 | .0550 | 4.90 | .0074 |
| .95 | .3867 | 2.95 | .0523 | 4.95 | .0071 |
| 1.00 | .3679 | 3.00 | .0498 | 5.00 | .0067 |
| 1.05 | .3499 | 3.05 | .0474 | 6.00 | .0025 |
| 1.10 | .3329 | 3.10 | .0450 | 7.00 | .0009 |
| 1.15 | .3166 | 3.15 | .0429 | 8.00 | .000335 |
| 1.20 | .3012 | 3.20 | .0408 | 9.00 | .000123 |
| 1.25 | .2865 | 3.25 | .0388 | 10.00 | .000045 |
| 1.30 | .2725 | 3.30 | .0369 | | |
| 1.35 | .2592 | 3.35 | .0351 | | |
| 1.40 | .2466 | 3.40 | .0334 | | |
| 1.45 | .2346 | 3.45 | .0317 | | |
| 1.50 | .2231 | 3.50 | .0302 | | |
| 1.55 | .2122 | 3.55 | .0287 | | |
| 1.60 | .2019 | 3.60 | .0273 | | |
| 1.65 | .1920 | 3.65 | .0260 | | |
| 1.70 | .1827 | 3.70 | .0247 | | |
| 1.75 | .1738 | 3.75 | .0235 | | |
| 1.80 | .1653 | 3.80 | .0224 | | |
| 1.85 | .1572 | 3.85 | .0213 | | |
| 1.90 | .1496 | 3.90 | .0202 | | |
| 1.95 | .1423 | 3.95 | .0193 | | |