請依照下列規則與格式，以橫式書寫方式將最終答案寫在彌封答案卷第一頁。並將各題之求解計算過程，由彌封答案卷第二頁開始書寫，並請務必註明題號。（第 8 題證明題由彌封答案卷第一頁開始寫）

　　1. 未按作答格式作答者，扣該科總分 10 分。

　　2. 未在彌封答案卷作答者，不予計分。

　　3. 無求解計算過程者，不予計分。

| |
| --- |
| 1(a). |
| 1(b). |
| 2 |
| 3. |
| 4. |
| 5. |
| 6. |
| 7(a). |
| 7(b). |
| 8(a). |
| 8(b). |

1. (5 + 5 points)

    (a) Find the derivative of $\dfrac{2x+1}{\cos 2x}$ at $x = \pi/2$.

    (b) Differentiate $f(x) = e^{2\ln(x^2+3)}$ to show that $f'(x) = ax^3 + bx$ and to find $(a,b)$.

2. (10 points) Evaluate $\displaystyle\int_{-2}^{2}\int_{-1}^{1} 3y^2 e^{xy}\,dx\,dy$

3. (5 + 5 points) Show that the line $x - y = 0$ is tangent to the curve given by the equation $y = x^3 - 8x^2 + 17x$. Find a point of tangency.

※ 注意：1.考生須在「彌封答案卷」上作答。
　　　　2.本試題紙空白部份可當稿紙使用。

科目：微積分　　　　　　　　　　　　系所組：企業管理學系管理學碩士班乙組

4. (10 points) Determine the given infinite geometric series converges or diverges. If it converges, find its sum.

$$\sum_{j=0}^{\infty} \frac{5^{j+1}}{6^j}$$

5. (10 points) A company is selling a product. If the consumer demand for the product is $D(p) = 1000e^{-0.02p}$, $p$ is the price. Find the price that maximizes the revenue.

6. (10 points) Find the volume under the surface $f(x,y) = 4xy + 3$ over the region shown the graph below.
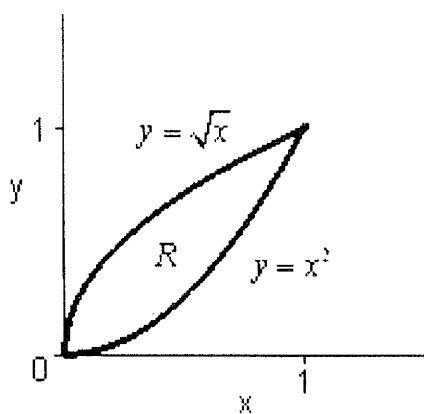


7. (10 + 10 points)

   (a) Let the rate of change of a population (at time $t$) is proportional to the population, i.e., $y'(t) = \alpha y(t)$. Denote $y(0) = A$, which is the initial population. Find $y(t)$ as a function of $t$ in terms of $\alpha$ and A.

   (b) Denote $y(t)$ the population of Taiwan in the year $t$ with $t =$ 西曆年.
   Suppose $y(1950) = 7\ million$ and $y(2010) = 23\ million$.
   Find $y(1990)$. (Round your answer to 1 decimal place.)

   Note: Use $\ln 7 = 1.9459$, $\ln 23 = 3.1355$, and the following exponential EXP(x) table (next page) to compute your answer if needed.

**Exponential Function (eˣ) Table** 〈$x = 2.60 \sim 2.99$〉

Example (how to use the table): $e^{2.60} = 13.4637$, $e^{2.61} = 13.5991$

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **2.6** | 13.4637 | 13.5991 | 13.7357 | 13.8738 | 14.0132 | 14.1540 | 14.2963 | 14.4400 | 14.5851 | 14.7317 |
| **2.7** | 14.8797 | 15.0293 | 15.1803 | 15.3329 | 15.4870 | 15.6426 | 15.7998 | 15.9586 | 16.1190 | 16.2810 |
| **2.8** | 16.4446 | 16.6099 | 16.7769 | 16.9455 | 17.1158 | 17.2878 | 17.4615 | 17.6370 | 17.8143 | 17.9933 |
| **2.9** | 18.1741 | 18.3568 | 18.5413 | 18.7276 | 18.9158 | 19.1060 | 19.2980 | 19.4919 | 19.6878 | 19.8857 |

8. (10 + 10 points)

(a) Define $\int_a^\infty f(x)dx = \lim_{b \to \infty} \int_a^b f(x)dx$.

Find $\int_1^\infty \frac{1}{x^2}dx = ?$ and $\int_1^\infty \frac{1}{\sqrt{x}}dx = ?$

(b) Prove or disprove the convergence of the following two series:

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$$

$$1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \cdots$$