(101)輔仁大學碩士班招生考試試題	考試日期：101 年 3 月 9 日第 二 節
	本試題共 貳 頁 (本頁為第 壹 頁)

科目：數學　　系所組：資訊工程學系

1.

(a) (5 pts.) Define the **partial order relation**.

(b) (7 pts.) Let $T=(V, E)$ be a rooted tree with root r. Define the relation R on V by xRy for $x, y \in V$ if $x=y$ or x is on the path from r to y. Prove R is a partial order.

2. Prove or disprove the following arguments.

(a) (5 pts.)

$$\begin{array}{l} a \vee b \\ a \rightarrow (c \rightarrow d) \\ m \rightarrow c \\ \neg b \\ \hline \therefore \neg d \rightarrow \neg m \end{array}$$

(b) (5 pts.)

$$\begin{array}{l} a \leftrightarrow b \\ b \rightarrow c \\ c \vee \neg d \\ \neg d \rightarrow b \\ \hline \therefore d \end{array}$$

3. Solve the recurrence relation $a_{n+2} - 5a_{n+1} + 6a_n = 2, n \geq 0, a_0 = 3, a_1 = 7$

(a) (4 pts.) by using the **characteristic equation**.

(b) (6 pts.) by applying the technique of **generating function**.

4.

(a) (4 pts.) Define the **Stirling number of the second kind**.

(b) (6 pts.) Let $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{w, x, y, z\}$. How many **onto functions** are there from A to B? (MUST calculate the value.)

5 (8 pts.) Let $G = (V, E)$ be a loop free, connected, not-multigraph, and planar graph with $|V| = v, |E| = e > 2$ and r regions. Then $3r \leq 2e$ and $e \leq 3v - 6$. Prove it.

※ 注意：1.考生須在「彌封答案卷」上作答。
2.本試題紙空白部份可當稿紙使用。
3.考生於作答時可否使用計算機、法典、字典或其他資料或工具，以簡章之規定為準。

6. Let A, B, S be three $n \times n$ matrices. If $B = S^{-1}AS$,
(a) Prove that A and B have the same eigenvalues. (5%)
(b) If x is an eigenvector of A corresponding to an eigenvalue λ, find an eigenvector of B corresponding also to λ. (5%)

7. Let $T : V \to W$ be a linear transformation and $v_1, \cdots, v_k \in V$. Prove or disprove the following statements:
(a) If $\{v_1, \cdots, v_k\}$ is linear independent then $\{T(v_1), \cdots, T(v_k)\}$ is still linear independent. (5%)
(b) If $\{v_1, \cdots, v_k\}$ is linear dependent then $\{T(v_1), \cdots, T(v_k)\}$ is still linear dependent. (5%)

8. Let W be the space spanned by $w_1 = (1, -2, -1, 2)$ and $w_2 = (-4, 1, 0, 3)$. Consider a vector $y = (3, -1, 1, 13)$
(a) Find the orthogonal projection of y onto W. (5%)
(b) Compute the distance from y to W. (5%)

9. Let

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{bmatrix}$$

Find the inverse of A by elementary row operations. (10%)

10. If P_2 denotes all polynomials of degree at most 2, and
$T : P_2 \to R$ is a linear transformation defined by

$$T(p) = \int_0^1 p(x)dx$$

Find a basis for the kernel of T. (10%)