

科目：工程數學(C)-線性代數 系所組：電機工程所 丙組

1. For the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix}$, find two nonsingular matrices P and Q such that PAQ is a diagonal matrix. (10%)

2. Given the system of linear equations: $x-2y+3z=2$, $2x+(k+1)y+6z=8$, $-x+3y+(k-2)z=-1$, determine the values of k such that: (10%)

(a) The system has infinitely many solutions.
 (b) The system has a unique solution.
 (c) The system has no solution.

3. Let $A = \begin{bmatrix} 3 & -1 & 1 \\ -2 & 1 & 1 \\ 1 & -1 & -2 \end{bmatrix}$, find the determinant of A , A^T , and A^{-1} . (10%)

4. Let $A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $A_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $A_3 = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$, $A_4 = \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix}$.

(a) Determine whether A_1 , A_2 , A_3 , and A_4 form a linearly dependent set or a linearly independent set. (10%)

(b) Is $A = \begin{bmatrix} 1 & 2 \\ -1 & 5 \end{bmatrix} \in \text{span}\{A_1, A_2, A_3, A_4\}$?, why? (5%)

5. $A = \begin{bmatrix} 1 & 0 & -2 & 3 \\ 2 & 2 & 0 & 4 \\ 2 & 0 & -4 & 6 \\ 1 & 1 & 1 & 1 \end{bmatrix}$, find $\text{rank}(A)=?$ (5%)

6. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 4 & 0 \\ 1 & 4 & 6 \\ 1 & 4 & 6 \end{bmatrix}$, use the Gram-Schmidt process to obtain an orthonormal basis for the column space of A . (10%)

※注意：1.考生須在「彌封答案卷」上作答。

2.本試題紙空白部分可當稿紙使用。

3.考生於作答時可否使用計算機、法典、字典或其他資料或工具，以簡章之規定為準。

7. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$. Use eigen-decomposition to diagonalize it into $A = S \Lambda S^{-1}$, where Λ is a diagonal matrix. Write down the eigenvalues, eigenvectors, and the matrices S and Λ . (15%)
8. A symmetric matrix A is said to be positive definite if $\bar{x}^T A \bar{x} > 0$ for all nonzero vectors \bar{x} . For what range of the number b is the following matrix positive definite? $A = \begin{bmatrix} 2 & 2 & 4 \\ 2 & b & 8 \\ 4 & 8 & 7 \end{bmatrix}$. (10%)
9. The quadratic equation $5x_1^2 - 4x_1x_2 + 5x_2^2 = 21$ is an ellipse. Please
- (A) use quadratic form of matrix representation to rewrite the equation, (3%)
 - (B) find its eigenvalues and eigenvectors, (5%)
 - (C) derive the length and angle of its major axis, (5%)
 - (D) plot out the curve. (2%)

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