科目: 工程數學(B)(微分方程·線性代數)系所組:電機工程(乙組)

1. Solve the initial value problem $\dfrac{dy(x)}{dx} + \dfrac{1}{5+x}y(x) = 2$; $y(1) = 1$. (15%)

2. Solve the given initial value problem (20%)

$$y'''(x) + 6y''(x) + 11y'(x) + 6y(x) = 9 - 30e^{2x};\ y(0) = 0,\ y'(0) = 0,\ y''(0) = 3.$$

3. Use the Laplace transform to solve the initial value problem shown below (15%)

$$y''(x) + 8xy'(x) - 16y(x) = 3;\ y(0) = y'(0) = 0.$$

4. Using only 0's and 1's, list all possible 2×2 matrices in row canonical form. (10%)

5. Find the rank, the rows of A forming a basis of the row space, and the null space of matrix A. (15%)

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 10 & -6 \\ 0 & -6 & 3 \\ -1 & -5 & 3 \end{bmatrix}$$

6. Find κ so that $x = [0, -12, -3, 8, 3, \kappa, 63]^{T}$, $y = [7, 5, -9, \kappa, -25, 19, 0]^{T}$ in $\mathbb{R}^{7}$ are orthogonal. (5%)

7. A matrix B is given below.

$$B = \begin{bmatrix} 4 & 3 \\ -2 & -1 \end{bmatrix}$$

(a) Find all eigenvalues and corresponding eigenvectors for matrix B. (8%)
(b) Find a matrix C such that C is nonsingular and CBC^{-1} is diagonal. (4%)
(c) Find an orthogonal matrix Q, and an upper triangular matrix R such that $B = QR$. (8%)