

科目：工程數學(A)(線性代數-機率學) 系所組：電子所甲組

1. The vectors are given in the following. Determine whether they are linearly independent. (12%)

a)  $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$  and  $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ , (6%)

(b)  $\begin{bmatrix} 1 \\ -1 \\ 2 \\ 3 \end{bmatrix}$ ,  $\begin{bmatrix} -2 \\ 3 \\ 1 \\ -2 \end{bmatrix}$ ,  $\begin{bmatrix} 1 \\ 0 \\ 7 \\ 7 \end{bmatrix}$  (6%)

2. If the factorization of matrix  $A$  into a product of a unit lower triangular matrix times a strictly upper triangular matrix  $U$ . (16%)a) Determine matrix  $L$  and  $U$ . (8%)b) Assume  $X$  is an  $3 \times 1$  vector. Let  $AX = B$ , determine  $X$  using the factorization method. (8%)

$$A = \begin{bmatrix} 2 & 4 & 2 \\ 1 & 5 & 2 \\ 4 & -1 & 9 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

3. Diagonalization (12%)

a)  $A = \begin{bmatrix} 2 & -3 \\ 2 & -5 \end{bmatrix}$ , (6%)

(b)  $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ , (6%)

4. Compute the following probabilities (20%)

a) A poker hand has at least two king. (5%)

b) A poker hand has 2 aces given that the first card is king. (5%)

c) Suppose particles arrive on the average once a second. Find the probability of at most 2 particles in the next 10 seconds. (5%)

d) Each bulb has an average lifetime of three months. Find the probability that the one bulb will last more than 6 months. Assume the lifetimes are exponential random variables. (5%)

5. Given the distribution function  $F(x)$ . (12%)a) Determine  $P[X=0]$  and  $P[0 < X \leq 1]$  from  $F(x)$ . (6%)b) Find and plot the pdf corresponding to the  $F(x)$ . (6%)

$$F(x) = \begin{cases} 0, & x \leq 0 \\ \frac{1}{2}x, & 0 \leq x < 1 \\ \frac{1}{2}, & 1 \leq x < 2 \\ 1, & x \geq 2 \end{cases}$$

※ 注意：1. 考生須在「彌封答案卷」上作答。

2. 本試題紙空白部分可當考稿紙使用。

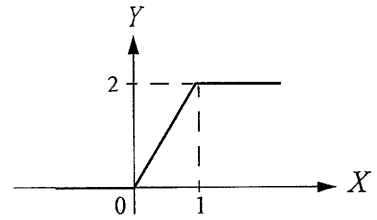
3. 考生於作答時可否使用計算機、法典、字典或其他資料或工具，以簡章之規定為準。

6. Let  $X$  be a random variable uniformly distributed in the interval  $[-2, 2]$

(13%)

a) Let  $Y = g(X)$ . Find  $P(Y \leq 1)$  from  $f(x)$ . (6%)

b) Find and plot the density of  $Y$ ,  $f(y)$ . (7%)

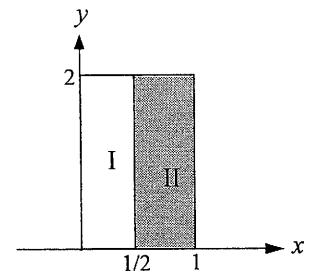


7. Let  $X$  and  $Y$  have the joint probability density function  $f_{XY}(x, y) = c$  in region I and  $f_{XY}(x, y) = 2c$  in region II.

(15%)

a) Find and plot the marginal distributions  $f_X(x)$  and  $f_Y(y)$ . (8%)

b) Determine  $P(|X - Y| \geq 0.5)$ . (7%)



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