(101)輔仁大學碩士班招生考試試題	考試日期：101 年 3 月 9 日第 4 節
	本試題共 1 頁 (本頁為第 1 頁)

科目： 近代物理　　系所組： 物理學系

1. (6%) Radiation of wavelength $\lambda = 290$ nm falls on a metal surface for which the work function is $W = 4.05$ eV. What potential is needed to stop the most energetic photoelectron? The Planck constant $h = 6.626 \times 10^{-34}$ J·s.

2. (9%) Calculate the de Broglie wavelength of
   (a) a mass of 1 g moving at a velocity of 1 m s^{-1}.
   (b) a free electron ($m_e = 9.1 \times 10^{-31}$ kg) having a kinetic energy of 200 eV.
   (c) a free proton having a kinetic energy of 200 eV.

3. (13%) Describe the setup of the Stern-Gerlach experiment and the important implication of the experiment results.

4. (24%) Consider a particle of mass m moving in a one-dimensional infinite square well.

$$V(x) = \begin{cases} \infty & x < 0 \\ 0 & 0 < x < a \\ \infty & x > a \end{cases}$$

   Consider a particle in the infinite potential well described by a wave function

$$\Psi(x, t = 0) = N \sin \frac{4\pi x}{a} \cos \frac{\pi x}{a}.$$

   (a) Determine the normalization N.
   (b) Determine $\Psi(x, t)$.
   (c) Compute the position expectation value $\langle x \rangle$ of this state $\Psi(x, t)$.

5. (24%) Consider a free particle of mass m. Given two wave functions representing two different quantum states at $t = 0$,

$$\psi(x) = e^{-\alpha x^2} \text{ and } \phi(x) = e^{\frac{ip_0 x}{\hbar} - \alpha x^2},$$

   where p_0 and α are positive real numbers.
   (a) Find the probability currents corresponding to these two states.
   (b) Find the energy expectation values for both states.
   (c) Describe qualitatively the differences between these two states.

6. (24%) Consider a two-state system and the state space is spanned by the orthonormal basis $\{|+\rangle, |-\rangle\}$. Given the following defintions of operators,

$$S_x = \frac{\hbar}{2}\left(|+\rangle\langle -| + |-\rangle\langle +|\right), \qquad S_y = \frac{i\hbar}{2}\left(-|+\rangle\langle -| + |-\rangle\langle +|\right),$$

$$S_z = \frac{\hbar}{2}\left(|+\rangle\langle +| - |-\rangle\langle -|\right).$$

   (a) Show that $[S_x, S_y] = i\hbar S_z$ and $[S_x, S_z] = -i\hbar S_y$.
   (b) Find the eigenvalues and eigenvectors of S_x.

Usefule formula: $\int_{-\infty}^{\infty} e^{-\gamma x^2} dx = \sqrt{\frac{\pi}{\gamma}}$ and $\int_{-\infty}^{\infty} dx\, x^2 e^{-\gamma x^2} dx = \frac{1}{2\gamma}\sqrt{\frac{\pi}{\gamma}}$.

※ 注意：1.考生須在「彌封答案卷」上作答。
2.本試題紙空白部份可當稿紙使用。
3.考生於作答時可否使用計算機、法典、字典或其他資料或工具，以簡章之規定為準。