科目： 應用數學　　　　　　　系所組： 物理學系碩士班

1. (a) Solve $y' = e^{2x} + y - 1$. **(10 %)**

   (b) Solve $y'' + 9y = 2\sec 3x$. **(10 %)**

2. (a) Evaluate $\oint_c \dfrac{1}{z(e^z - 1)} dz$, C: $|z| = 1$, counterclockwise. **(10 %)**

   (b) Evaluate $\displaystyle\int_0^\infty \dfrac{2\cos^2 x}{(4 + x^2)^2} dx$. **(15 %)**

3. Find the inverse Laplace transform of $F(s) = \dfrac{2s + 1}{s^2 + 2s + 5}(1 - e^{-\pi s})$. **(10 %)**

4. The solution of the problem

   $$\alpha^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad 0 \le x \le L, \quad t \ge 0$$

   $$u(0, t) = 0, \quad \left.\frac{\partial u(x, t)}{\partial x}\right|_{x=L} = 0, \quad t \ge 0$$

   can be represented by $u(x, t) = \sum_n c_n u_n(x, t) = \sum_c c_n X_n(x) T_n(t)$.

   Find the separated-variable solution $u_n(x, t)$. **(15 %)**

5. Find the Fourier transform of $f(t) = \displaystyle\int_{-\infty}^\infty [e^{-a(t-\tau)^2}][\sum_{m=-\infty}^\infty \delta(\tau - mT)] d\tau$, where a and T are constants, and $\delta(\cdot)$ is the Dirac delta function. **(15 %)**

6. Find the series solution about $x = 0$ for $2x^2 y'' - xy' + (1 + x)y = 0$. **(15 %)**