(101)輔仁大學碩士班招生考試試題	考試日期：101 年 3 月 9 日第 3 節
	本試題共 — 頁 (本頁為第 頁)

科目： 高等微積分 　　　　系所組： 數學

1. (10 points) Find $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{k^2}\sin\frac{1}{k}$. (*Hint.* Using the Riemann sum to $x\sin x$ on $[0,1]$)

2. (15 points) Let $f(x)$ and $g(x)$ be continuous functions on the closed interval $[-1,1]$. Show that there is a point x_0 in the open interval $(-1,1)$ such that $f(x_0)=g(x_0)$ if $f(-1)=1=g(1)$ and $f(1)=-1=g(-1)$. (*Hint.* Intermediate value theorem for continuous functions)

3. (15 points) Let $f(x)=\dfrac{\sin x}{x}$ whenever $x\neq 0$ and $f(0)=1$. Show that the function $f(x)$ is uniformly continuous on the whole real line $\mathcal{R}$.

4. (15 points) Let

$$f(x)=\begin{cases} xe^{-x^2}, & x>0,\\ 1, & x<0,\\ \frac{1}{2}, & x=0\end{cases}$$

and define

$$F(x)=\int_0^x f(t)dt.$$

Prove or disprove that $F(x)$ is differentiable at $x=0$.

5. (15 points) A sequence is defined by $x_1=2$ and

$$x_{n+1}=\frac{5+x_n^2}{2x_n},\quad n=1,2,\ldots.$$

Show that the sequence of numbers $\{x_n\}_{n=1}^{\infty}$ has a limit.

6. (15 points) Show the sequences of functions $\{x^n\}_{n=0}^{\infty}$ converges uniformly on $[-\frac{\delta}{2},\delta]$ where $0<\delta<1$ and this sequence is not uniformly convergent on $[-\frac{1}{2},1]$.

7. (15 points) Let

$$\begin{aligned} u &= f(x,y)=ax+by+u_0,\\ v &= g(x,y)=cx+dy+v_0,\end{aligned}$$

where a, b, c, d, u_0, and v_0 are constants. The variables (x,y) can be represent by the variables (u,v) if the matrix

$$J=\begin{bmatrix} a & b\\ c & d\end{bmatrix}=\begin{bmatrix} f_x & f_y\\ g_x & g_y\end{bmatrix}$$

has an inverse. Assume that

$$\begin{aligned} u &= T(x,y)=x^2+xy^2,\\ v &= S(x,y)=x^2y+y^2\end{aligned}$$

and we know that $T(1,2)=5$ and $S(1,2)=6$. Can the variables (x,y) be represented locally by the variables (u,v) at $(u,v)=(5,6)$? State your reason.

※ 注意：1.考生須在「彌封答案卷」上作答。
2.本試題紙空白部份可當稿紙使用。
3.考生於作答時可否使用計算機、法典、字典或其他資料或工具，以簡章之規定為準。