東吳大學 101 學年度碩士班研究生招生考試試題

系級	財務工程與精算數學系碩士班 A、B 組	考試時間	100 分鐘
科目	微積分	本科總分	100 分

1. (10%) Evaluate the limit.

$$\lim_{h\to 0}\frac{2\sqrt[4]{16+h}-4}{h}$$

2. (10%) Find the values of x at which

$$F(x)=\int_0^{x^2}\frac{t^2-5t+4}{t^2+1}dt$$

has relative extrema.

3. (10%) Prove that

$$0\le\int_0^1\frac{x^5}{\sqrt[3]{1+x^4}}dx\le\frac{1}{6}.$$

4. (10%) Find all value(s) of a such that f is continuous on $\mathbf{R}$:

$$f(x)=\begin{cases}x+1 & \text{if } x\le a\\ x^2 & \text{if } x>a\end{cases}$$

5. (10%) Find the directional derivative of $f(x,y)=e^x\cos(2y)$ at the point $(0,\frac{\pi}{4})$ in the direction of $\mathbf{v}=2\mathbf{i}+3\mathbf{j}$.

6. (10%) Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty}\frac{n!}{n^n}$$

7. (10%) Evaluate the limit.

$$\lim_{n\to+\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{n+n}\right).$$

8. (10%) Find the points closest to the origin (原點) on the surface $x^2-z^2-1=0$.

9. (10%) Evaluate the indefinite integral.

$$\int\sec^3 x\,dx$$

10. (10%) Find the area of the region cut from the first quadrant by the cardioid (心臟線) $r=1+\sin\theta$.