

系級	財務工程與精算數學系碩士班 A、B 組	考試時間	100 分鐘
科目	機率統計	本科總分	100 分

- Let X be a random variable with pdf $f_X(x) = \begin{cases} 4x^3, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$. Determine the pdf of the following random variables:
 (a) $Y = \ln X$. (b) $Z = (X - 0.5)^2$. (10+10 pts)
- Let $X \sim \text{binomial}(n, p)$ and $\hat{p} = \frac{X}{n}$.
 (a) Find a constant c so that $E[c \hat{p}(1 - \hat{p})] = p(1 - p)$. (5 pts)
 (b) Find an unbiased estimator of $\text{Var}(X)$. (5 pts)
 (c) Consider a random sample X_1, \dots, X_{100} from binomial (n, p) . Find unbiased estimator of p and $\text{Var}(X)$ based on the random sample. (5+5pts)
- Assume that passengers arrive at a bus station in a Poisson process with rate λ (per minute).
 (a) Given that there are $m+n$ arrivals within the time interval (2:00pm, 3:00pm), find the probability that there were m arrivals between 2:00 and 2:10. (10 pts)
 (b) If the station opens at 6:00am, find the probability that the third passenger arrives after 6:05am. (10 pts)
- Let D represent the annual demand for some commodity that is measured in gallons (or fractions thereof). At the beginning of the year, a supply store order c gallons at 6 dollars per gallon and sell it to customers at 14 dollars per gallon. If $D \sim \text{uniform}(980, 1020)$,
 (a) find the expected amount sold. (10 pts)
 (a) find the optimum choice for c . (10 pts)
- Let X_1, \dots, X_{100} be a random sample from exponential distribution with mean 2. Find
 (a) $P(\min(X_1, X_2) < 2)$ (b) $P(\max(X_1, X_2) < 2)$ (5+5 pts)
 (c) $P(\frac{X_1 + X_2}{2} < 2)$ (d) $P(\frac{X_1 + \dots + X_{100}}{100} < 2)$ (5+5 pts)