國立高雄大學 101 學年度研究所碩士班招生考試試題

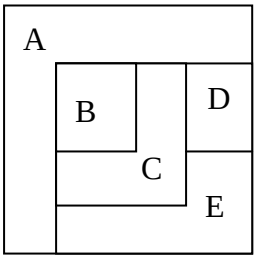
科目：離散數學與資料結構　　系所：資訊工程學系　　是否使用計算機：否
考試時間：100 分鐘　　本科原始成績：100 分

1. In an election, there were totally 11,999 votes. Five candidates ran for this election, and the numbers of votes were a_1, a_2, a_3, a_4, and a_5 with $a_1 > a_2 > a_3 > a_4 > a_5$.

(a) (3%) What is the minimum of a_1?
(b) (3%) What is the maximum of a_3?
(c) (3%) If a_1 is not divisible by 2 or 3, what is the number of possible a_1 counts?
(d) (3%) Let $S_{i,j} = a_i + a_{i+1} + \cdots + a_j$, $i \le j$. Show that there exists one $S_{i,j}$ such that $S_{i,j}$ is divisible by 4.

2. John has to take 8 courses: Programming I, Programming II, Calculus I, Calculus II, English I, English II, Probability I, and Probability II. The precedence requirements for these 8 courses are Programming I before Programming II, Calculus I before Calculus II, English I before English II, and Probability I before Probability II.
(a) (2%) What is a topological sort?
(b) (2%) What is the number of possible results of a topological sort for these 8 courses?
(c) (3%) If John wants to take these 8 courses in 5 semesters, what is the number of possible arrangements in taking these 8 courses?
(d) (3%) If John wants to take these 8 courses in 4 semesters and at least one course for each semester, what is the number of possible arrangements in taking these 8 courses?
(e) (3%) If there are more precedence requirements: Programming I before Calculus I, Calculus I before English I, English I before Probability I, Programming II before Calculus II, Calculus II before English 1I, and English II before Probability II, what is the number of possible results of a topological sort for these 8 courses?

3. Mary wants to paint the following 5 countries such that any two neighboring countries cannot be painted in a same color.



(a) (3%) Plot the relations of these 5 countries in a graph G such that the 5 vertices are A, B, C, D, and E, and the edges are (x, y), $x \neq y$, that x and y are neighbors.

(b) (2%) What is the chromatic number of the graph G in (a)?
(c) (3%) What is the chromatic polynomial of the graph G in (a)?
(d) (3%) What is the number of possible colorings if 6 colors are used?

4. There are five chairs put in a row and numbered 1, 2, 3, 4, and 5 from left to right for five children, Tom, John, Mary, Grace, and Brian that each child sits on a chair.
(a) (3%) If Tom cannot be seated at chair 1, John cannot be seated at chair 2, Mary cannot be seated at chair 3, Grace cannot be seated at chair 4, and Brian cannot be seated at chair 5, what is the number of possible seating arrangements?
(b) (3%) If Tom cannot be seated next to John and Mary cannot be seated next to Grace, what is the number of possible seating arrangements?
(c) (3%) If Tom cannot be seated at chair 3, John cannot be seated at chair 3, Mary cannot be seated at chair 2 or chair 4, Grace cannot be seated at chair 3, and Brian cannot be seated at chair 5, what is the number of possible seating arrangements?

5. (5%) Find $1^2 + 2^2 + 3^2 + \cdots + n^2$ by solving $a_n = a_{n-1} + n^2$, $a_1 = 1$, $n \geq 2$.

6. Circular doubly linked list coding problem.
(a) Write the code segment that deletes the node to which x points from a circular doubly linked list. (5%)
(b) Write the code segment that inserts into a circular doubly linked list the node to which y points before the node to which x points. (5%)

7. Consider a string you received:

lossless

(a) Construct its Huffman tree. (5%)
(b) Generate its Huffman code. (3%)
(c) How many bits are required to encode the above string ? (2%)
(d) Assume you receive one more character of '**s**'. Please construct the new Huffman tree. (3%)

8. Given a non-directed graph below, in which each number on the edges represents the edge cost, and each number in the circles represents the vertex index.

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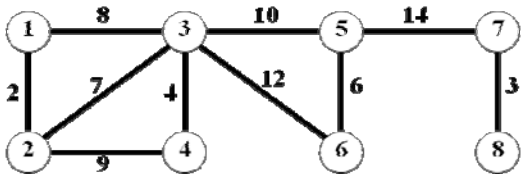
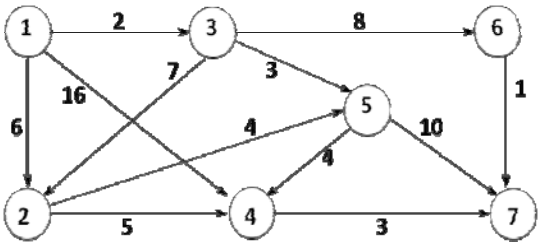


(a) Show how to use Kruskal's method to generate the minimum cost spanning tree. Show the running steps clearly. (5%)
(b) What is the corresponding minimum cost? (2%)

9. Given a directed graph below, in which each number on the edges represents the edge cost, and each number in the circles represents the vertex index. Show how to use Dijkstra's algorithm to find the shortest path from vertex 1 to vertex 7 in the graph. Show the running steps clearly. (10%)



10. Input a sequence of data in the order from left to right: 4, 2, 8, 5, 3, 6, 7, 1, 9
(a) Construct the binary search tree step-by-step. (5%)
(b) Continued from (a), construct the AVL tree step-by-step. (5%)