系所：

科目：工程數學　　　　　　土木與環境工程學系(土木工程組)　　是否使用計算機：是

考試時間：100 分鐘

本科原始成績：100 分

1. Show $y = (c - e^x)/(2x)$ (where c is a constant) is the solution to the differential equation, $y' = -(2y + e^x)/(2x)$ (10%)

2. Find the constants c_1 and c_2 for $y = c_1 e^x + c_2 e^{4x}$, the solution of the initial value problem: $y'' - 5y' + 4y = 0; y(0) = 1, y'(0) = -4$ (10%)

3. $y = \sum_{n=0}^{\infty} a_n x^n$ is the solution of $y'' + xy = 0$. Find the first 5 non-zero items if $a_0 = 1$ and $a_1 = 0$ (10%)

4. Find the eigenfunction(s) for the boundary value problem: $y'' + \lambda y = 0$ (where λ is a constant); $y'(0) = y'(2\pi) = 0$ (10%)

5. Solve the initial value problem: $y'' + 11y' + 24y = 2u(t - \pi); y(0) = 1, y'(0) = -3$ (10%)

6. Determine the value of s to make the following four points to be on the same plane: A(0, -1, -1) , B(4, 5, s), C(3, 9, 4), D(-4, 4, 4). (10%)

7. A = $\begin{bmatrix} 2 & 4 & -6 \\ 4 & 2 & -6 \\ -6 & -6 & -15 \end{bmatrix}$, there exists a diagonal matrix $D = P^T A P$, Determine P=? and D=? (10%)

8. For which condition of λ values and corresponding rank is the following system with and without solutions? (10%)

$$x_1 + 2x_2 + 3x_3 = 1$$
$$4x_1 + 5x_2 + 6x_3 = 4$$
$$9x_1 + 8x_2 + 7x_3 = \lambda$$

9. Find the determinant of the following matrix (10%)

$$A = \begin{bmatrix} 1 + a_1 & a_2 & \cdots & a_n \\ a_1 & 1 + a_2 & \cdots & a_n \\ a_1 & a_2 & \ddots & \vdots \\ a_1 & a_2 & \cdots & 1 + a_n \end{bmatrix}$$

10. A fluid with the velocity field of $v = x\vec{i} + y\vec{j} + 0\vec{k}$. Show whether (a) the fluid is compressible, (b) the flowing field is rotational? (10%)