國立高雄大學 101 學年度研究所碩士班招生考試試題

科目：工程數學
考試時間：100 分鐘

系所：
電機工程學系(通訊組)
本科原始成績：100 分

是否使用計算機：是

共十題，每題十分。請依題號順序作答，否則酌予扣分。

1. Determine the kernel and range of each of the following linear transformations from R^3 into R^3.

   (a) $L(\mathbf{x}) = (x_1,\ x_2,\ 0)^T$.

   (b) $L(\mathbf{x}) = (x_1,\ x_1,\ x_1)^T$.

2. Find the **QR**-decomposition of **A** under the Euclidean inner product.

$$\mathbf{A} = \begin{bmatrix} 2 & 0 & 2 \\ 2 & 2 & -1 \\ 1 & -1 & -2 \end{bmatrix}$$

3. Find a matrix **P** that orthogonally diagonalizes **A**, and determine $\mathbf{P}^{-1}\mathbf{AP}$.

$$\mathbf{A} = \begin{bmatrix} -1 & 2 & 0 \\ 2 & 0 & 2 \\ 0 & 2 & 1 \end{bmatrix}$$

4. Let $T : R^3 \to R^3$ be defined by $T(x_1, x_2, x_3) = (x_1 - x_2, x_2 - x_1, x_1 - x_3)$.

   (a) Find the matrix for T with respect to the basis $B = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, where

$$\mathbf{v}_1 = (1,\ 0,\ 1),\ \ \mathbf{v}_2 = (0,\ 1,\ 1),\ \ \mathbf{v}_3 = (1,\ 1,\ 0)$$

   (b) Find $[T(\mathbf{x})]_B$ where $\mathbf{x} = (x_1,\ x_2,\ x_3)$ in R^3.

5. Let $B = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ be a basis for a vector space V and $T : V \to V$ the linear operator for which

$$T(\mathbf{v}_1) = \mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3 + 3\mathbf{v}_4$$
$$T(\mathbf{v}_2) = \mathbf{v}_1 - \mathbf{v}_2 + 2\mathbf{v}_3 + 2\mathbf{v}_4$$
$$T(\mathbf{v}_3) = 2\mathbf{v}_1 - 4\mathbf{v}_2 + 5\mathbf{v}_3 + 3\mathbf{v}_4$$
$$T(\mathbf{v}_4) = -2\mathbf{v}_1 + 6\mathbf{v}_2 - 6\mathbf{v}_3 - 2\mathbf{v}_4$$

   (a) Find the rank and nullity of T.

   (b) Determine whether T is one-to-one.

背面尚有試題

科目：工程數學
考試時間：100 分鐘

系所：
電機工程學系(通訊組)
本科原始成績：100 分

是否使用計算機：是

6. Let X be a random number from (0,1). Find the probability density function of Y=1/X.

7. Experience shows that X, the number of customers entering a postoffice, during any period of length t, is a random variable whose probability function is of the form

$$p(i) = k\frac{(2t)^i}{i!}, \quad i = 0,1,2,\cdots$$

(a) Determine the value of k.

(b) Compute $P(X < 4)$ and $P(X > 1)$.

8. The joint probability density function of random variables X and Y is given by

$$f(x,y) = \begin{cases} \lambda xy^2 & 0 \le x \le y \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Determine the value of λ.

(b) Find the marginal probability density functions of X and Y.

9. Let the random variables W and Z be defined by

$W = \min(X,Y)$ and $Z = \max(X,Y)$.

Find the joint cdf of W and Z in terms of the joint cdf of X and Y.

10. Suppose U and V are independent zero-mean, unit-variance Gaussian random variables, and let

$$X = U + V \qquad Y = 2U + V$$

Find the joint characteristic function of X and Y, and find $E[XY]$.