國立東華大學招生考試試題

招生學年度	101	招生類別	碩士班
系所班別	應用數學系統計碩士班		
科目	機率與統計		
注意事項	本考科禁止使用掌上型計算機；含機率論與統計學		

Note: The exam has 6 questions, for a total of 100 points. Explain your answer and write down necessary details of your calculation. No explanation/details = No credits.

1. *True or False?* In the following questions, determine whether the statement is true or false. Justify/Explain your answer. Give an example or explain briefly if the answer is "True", otherwise, give a counterexample.

(5) (a) If A, B are two disjoint events and $P(A) > 0, P(B) > 0$ then A and B are independent.

(5) (b) If X is a continuous random variable with pdf (probability density function) f then $P(X = x) = f(x)$ for all x.

(5) (c) If X, Y are independent then $E(\frac{X}{Y^2}) = \frac{E(X)}{E(Y^2)}$ provided all these expectations exist.

For question d), e), f), assume that $X_1, X_2, ..., X_n \sim_{iid} f_\theta(x)$ with $\theta \in \mathcal{R} = (-\infty, \infty)$ and $X = (X_1, \cdots, X_n)'$.

(5) (d) If a statistic $(T(X))^2$ is sufficient for θ then $T(X)$ is also sufficient for θ.

(5) (e) If $\delta(X)$ is a uniformly minimum variance unbiased estimator (UMVUE) for θ and $Var(\delta(X)) < \infty$ then $\delta(X)$ has the smallest mean square error among all unbiased estimators.

(5) (f) If $T(X)$ is an unbiased maximum likelihood estimator (MLE) for θ then $(T(X))^2$ is an unbiased MLE for θ^2.

(10) 2. Show that

$$\sum_{k=0}^{n} \binom{n}{k} (-2)^k = (-1)^n.$$

(10) 3. Let X has a discrete pmf (probability mass function) $f(x|\theta), \theta \in \Theta = \{-1, 1\}$ given below
(For example, $f(1|-1) = 0.3, f(-1|1) = 0.3$.)

	θ	
x	-1	1
1	0.3	0.3
0	0.5	0.4
-1	0.2	0.3

Find MLE of θ^2. Is this MLE unbiased? Justify your answer.

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4. Let X, Y have the joint pdf

$$f(x) = \begin{cases} cxy & \text{if } 0 < x < 1 \text{ and } 0 < y < x, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

(10) (a) Determine c such that f defines a pdf

(10) (b) Compute $P(Y + X < 1 \,|Y < 1/2)$.

5. Let $X_1, \cdots, X_n \sim_{iid} Poisson(\lambda)$ with $\lambda > 0$ and its pmf

$$f_\lambda(x) = \begin{cases} e^{-\lambda} \frac{\lambda^x}{x!}, & \text{if } x = 0, 1, \cdots; \\ 0 & \text{otherwise.} \end{cases}$$

(10) (a) Find a complete sufficient statistic for λ.

(10) (b) Find a UMVUE for λ. Is Cramér-Rao lower bound (CRLB) attainable in this case?

(10) 6. Let $X_1, X_2, ..., X_n \sim_{iid} N(\theta, \sigma^2)$ where $\theta \in \mathcal{R}$ and σ^2 is known.
Show that there is no UMP (uniformly most powerful) level α test for testing

$$H_o : \theta = 0 \quad versus \quad H_1 : \theta \neq 0$$

for any $\alpha \in (0, 1)$.