

元智大學 103 學年度研究所 碩士班 招生試題卷

系(所)別： 通訊工程學系碩士班 組別： 通訊組 科目： 工程數學 用紙第 / 頁共 2 頁

●不可使用電子計算機

1. (10%) (Probability Theory) With your own words, please explain:
 - (a) *Random variable.* (5%)
 - (b) *Independent events.* (5%)

2. (20%) The probability density function of random variable X is

$$f_X(x) = \begin{cases} 0.5e^{-x/2} & x \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

What is the second moment of X ?

3. (20%) Suppose that X is a Gaussian random variable with mean μ and variance σ^2 , and let $Z = aX + \beta$, where a (nonzero) and β are scalars. Please find the probability density function of Z .

4. (20%) Assume that

$$C = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$$

where ρ is a coefficient, which satisfies $|\rho| \leq 1$.

- (a) Please find the eigenvalues (λ) and eigenvectors (\mathbf{v}). (8%)
- (b) If $\rho > 0$, please find the eigenvector in (a) which can maximize $\mathbf{v}^T C^{-1} \mathbf{v}$ and determine the maximum value of $\mathbf{v}^T C^{-1} \mathbf{v}$. (6%)
- (c) If $\rho < 0$, please find the eigenvector in (a) which can maximize $\mathbf{v}^T C^{-1} \mathbf{v}$ and determine the maximum value of $\mathbf{v}^T C^{-1} \mathbf{v}$. (6%)

5. (10%) Consider the $(2M+1) \times 2$ matrix and M being a positive integer

$$H = \begin{bmatrix} 1 & -M \\ 1 & -(M-1) \\ \vdots & \vdots \\ 1 & M \end{bmatrix}$$

- (a) Prove the columns of H are linearly independent. (5%)
- (b) Please find the inverse of $H^T H$. (5%)

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6. (20%) Let

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 1 & 3 & 4 \end{bmatrix}$$

Find the bases for $R(A)$, $N(A)$, $R(A^T)$, and $N(A^T)$.

($R(A)$ and $R(A^T)$ are the range spaces of A and A^T , respectively.)

($N(A)$ and $N(A^T)$ are the null spaces of A and A^T , respectively.)

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