元智大學 103 學年度研究所 **碩士班** 招生試題卷

系(所)別：通訊工程學系碩士班　組別：微波組　科目：工程數學　用紙第 1 頁共 1 頁

●不可使用電子計算機

1. (Linear Algebra A: 15%, B: 15%) Total: 30%

A. Find the **eigenvalues** and **eigenvectors** of the matrix

$$A=\begin{bmatrix} 7.3 & 0.2 & -3.7 \\ -11.5 & 1.0 & 5.5 \\ 17.7 & 1.8 & -9.3 \end{bmatrix}$$

B. **Diagonalize** the matrix A and show the detail of your work.

2. (Differential Equation 20%)

Solve the following non-homogeneous linear ordinary differential equation (ODE)

$$x^2y''-3xy'+3y=x\ln x$$

3. (Laplace Transforms 20%)

Solve the following initial value problems (IVP) by using the **Laplace Transformation**.

$$xy''+(4x-2)y'-4y=0,\quad y(0)=1\quad y'(0)=0$$

4. (Fourier Analysis 20%)

Find the **Fourier transform** of the following function by using the definition,

$\Im\{f(x)\}=F(\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(x)e^{-i\omega x}dx$,

$f(x)=e^{-a|x|}$ where a is a positive real number.

5. (Vector Analysis 10%)

Let $f(x,y,z)=zy+yx$ and $\vec{v}=[v_x(x,y,z),v_y(x,y,z),v_z(x,y,z)]=[y^2,z^2,x^2]$.

Find the following:

A. (5%) The **curl** of the **gradient** of f, $\nabla\times(\nabla f(x,y,z))$.

B. (5%) The **divergence** of the **curl** of $\vec{v}$, $\nabla\cdot(\nabla\times\vec{v}(x,y,z))$.

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