

# 元智大學 103 學年度研究所 碩士班 招生試題卷

系(所)別： 機械工程學系碩  
士班

組別： 甲組

科目： 工程數學

用紙第 1 頁共 2 頁

●不可使用電子計算機 (盡量作答，請勿空白)

1. Solve the initial value problem. (16%)

$$y' = -\frac{y}{x} \quad \text{with} \quad y(1) = 1$$

2. Using the method of Laplace Transformation to solve the initial value problem of  $y(t)$ . (17%)

$$y'' + 2y' + y = e^{-t} \quad \text{with} \quad y(0) = -1, \quad \left. \frac{dy}{dt} \right|_{t=0} = 1$$

3. Solve the linear system. (10%)

$$\begin{aligned}x_1 - x_2 + x_3 &= 0 \\-x_1 + x_2 - x_3 &= 0 \\10x_2 + 25x_3 &= 90 \\20x_1 + 10x_2 &= 80\end{aligned}$$

4. Find the directional derivative of  $f$  at  $P$  in the direction of  $\vec{a}$ . (10%)

(a)  $f = xyz$ ,  $P: (-1, 1, 3)$ ,  $\vec{a} = [1, -2, 2]$  (5%)

(b)  $f = e^x \sin y$ ,  $P: (2, \pi/2, 0)$ ,  $\vec{a} = [2, 3, 0]$  (5%)

5. For the matrix  $A = \begin{bmatrix} 5 & -1 & 0 \\ -1 & 5 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ , answer the following questions. (13%)

(a) Find the eigenvalues of  $A$ . (5%)

(b) Find an orthogonal matrix  $P$  such that  $P^T A P$  is diagonal. (8%)

**103001**

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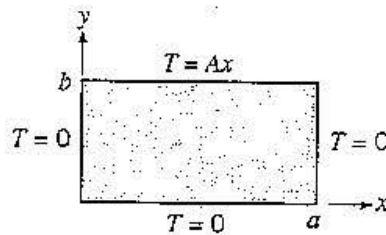
組別： 甲組

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用紙第 2 頁共 2 頁

●不可使用電子計算機

6. A two-dimensional rectangular plate is subjected to the boundary conditions shown as below. Derive an expression for the steady state temperature distributions  $T(x,y)$  with solving the heat conduction equation. (17%)



The heat conduction equation is :

$$\frac{\partial^2 T(x,y)}{\partial x^2} + \frac{\partial^2 T(x,y)}{\partial y^2} = 0$$

Please find the solution in sin, cos, sinh, cosh series functions by the method of separation variables.

7. There is periodic square wave with analytic represented as  $f(x)$  function

$$f(x) = \begin{cases} -k & \text{when } -\pi < x < 0 \\ k & \text{when } 0 < x < \pi \end{cases}$$

and  $f(x+2\pi) = f(x)$

Please find the Fourier coefficient of  $a_n, b_n$  and their series functions to present the  $f(x)$  functions. (17%)

**103002**