

元智大學 103 學年度研究所 碩士班 招生試題卷

系(所)別：工業工程與管理
學系碩士班

組別：不分組

科目：機率與統計

用紙第 1 頁共 1 頁

●不可使用電子計算機

- (7+4+4 points) A batch of 100 containers for frozen orange juice contains 5 that are defective. Two are selected, at random, without replacement from the batch. Let A and B denote the events that are first and second container selected is defective, respectively.
 - Compute $P(A)$ and $P(B)$.
 - Are A and B independent events? Why or why not?
 - If the sampling were done with replacement, would A and B be independent? Why or why not?
- (5+5+10 points) A secretary typed 5 letters and addressed 5 envelopes. For some reason the letters were put into envelopes at random. Let L_i be the event that i th, $i = 1, 2, \dots, 5$, letter is placed in the correct envelope.
 - Compute $P(L_1)$.
 - Compute $P(L_1 \cap L_2)$.
 - Compute the probability of at least one letter being put into the correct envelope.
- (3+5+7 points) Let random variables be such that $E(X) = E(Y) = 0$, $\text{Var}(X) = \text{Var}(Y) = 1$, and correlation coefficient $\rho_{X,Y} = \rho$. If $W = X - \rho Y$.
 - find $E(W)$.
 - Find $\text{Var}(W)$.
 - Find $\rho_{W,Y}$.
- (14+8+8 points)
 - Suppose we should use a paired- t test to compare the difference between two population means, however, a 2-sample t test is considered by mistake. Comment on the consequence risk for such mistake.
 - When performing a hypothesis testing, we conclude at a significant level α . Why do we need to pre-assign a significant level α in the hypothesis testing?
 - Is P -value referring to as a Type I error rate or a Type II error rate? Why? What practical phenomenon can we observe from P -value?
- (5+5+10 points) It is known that 20% of a brand of energy saving light bulbs will burn out before their guarantee has expired. If 900 bulbs are sold, let Y be the number of original bulbs that must be replaced.
 - Find the expected value of Y .
 - Find the variance of Y .
 - Within what limits would Y be expected to fall? Why?