

科目：微積分

系所組：統資系應用統計碩士班

一、Evaluate: (64%)

(a) $\lim_{x \rightarrow \infty} (\sqrt{x^3 + 15x + 20} - \sqrt{x^3 - 5x + 10})$

(b) $\lim_{x \rightarrow 0} \frac{e^x + e^{-x} - 2}{1 - \cos 3x}$

(c) Find $\frac{dy}{dx}$ at $(1, -2)$ if $y^4 + 3y = 4x^3 + 5x + 1$.

(d) Find $f'(1)$ if $f(x) = \int_0^{2x} (t^3 + 1)^{10} dt$.

(e) $\int \frac{1 + \sqrt{x}}{1 - \sqrt{x}} dx$

(f) $\int \ln(x + 1) dx$

(g) $\int_0^4 e^{-\sqrt{x}} dx$

(h) $\iint_R \frac{y}{x^3 + 2} dA$, where R is the region in the xy -plane bounded by the lines $x = 1$, $y = 0$, and $y = x$.

二、Let $f(x) = \frac{1}{10}(x^4 - 4x^3)$. (a) Find the relative maxima and relative minima of f . (b) Find the points of inflection of f . (c) Sketch the graph of f . (12%)

三、Phillip, the proprietor of a vineyard, estimates that the first 10,000 bottles of wine produced this season will fetch a profit of \$5/bottle. But if more than 10,000 bottles were produced, then the profit per bottle for the entire lot would drop by \$0.0002 for each additional bottle sold. Assuming that at least 10,000 bottles of wine are produced and sold, what is the maximum profit? (12%)

四、Suppose an investment is expected to generate income at the rate of $f(t) = 3000 + 80t$ dollars/year for the next 10 years. Find the present value of the income from this investment if the prevailing interest rate is 5%/year compounded continuously. (12%)

※ 注意：1. 考生須在「彌封答案卷」上作答。

2. 本試題紙空白部份可當稿紙使用。

3. 考生於作答時可否使用計算機、法典、字典或其他資料或工具，以簡章之規定為準。