國　立　宜　蘭　大　學

１０１學年度研究所碩士班考試入學

離散數學試題

（電子工程學系碩士班）

准考證號碼：

《作答注意事項》

1. 請先檢查准考證號碼、座位號碼及答案卷號碼是否相符。
2. 考試時間：100 分鐘。
3. 本試卷共有 10 題，一大題 10 分，共計 100 分。
4. 請將答案寫在答案卷上。
5. 考試中禁止使用大哥大或其他通信設備。
6. 考試後，請將試題卷及答案卷一併繳交。
7. 本試卷採雙面影印，請勿漏答。
8. 應試時不得使用電子計算機。

Question 1

Let $\emptyset$ be the empty set. Determine whether each of the following statements is TRUE or FALSE. [10 marks]

(a) $\emptyset \in \{\emptyset\}$. [2 marks]

(b) $\emptyset \subseteq \{\emptyset\}$. [2 marks]

(c) $\emptyset \cap \{\emptyset\} = \emptyset$. [2 marks]

(d) $\emptyset \cup \{\emptyset\} = \emptyset$. [2 marks]

(e) $\{\emptyset\} - \emptyset = \emptyset$. [2 marks]

Question 2

Define a binary relation $\mathcal{R}$ on the set $\mathbb{Z}$ of all the integers by $a\mathcal{R}b$ if and only if $a = |b|$. Determine whether each of the following statements is TRUE or FALSE.

(a) $\mathcal{R}$ is a reflexive relation. [2 marks]

(b) $\mathcal{R}$ is a symmetric relation. [2 marks]

(c) $\mathcal{R}$ is an antisymmetric relation. [2 marks]

(d) $\mathcal{R}$ is a transitive relation. [2 marks]

(e) $\mathcal{R}$ is an equivalence relation. [2 marks]

Question 3

Answer the following questions briefly.

(1) Compute the value of $3^{100} \mod 4$. [5 marks]

(2) Compute the value of $1 + 3 + 3^2 + \cdots + 3^{100} \mod 4$. [5 marks]

Question 4

What is the coefficient of $x_1^3 x_2 x_3^2$ in the expansion of $(2x_1 - 3x_2 + 5x_3)^6$? [10 marks]

Question 5

Answer the following questions briefly.

(1) At a party there are 6 men and 6 women. In how many ways can the 6 women choose male partners for the first dance ? [5 marks]

(2) How many ways are there for the second dance if everyone has to change partners? [5 marks]

背面尚有試題

Question 6

Let *n* be a positive integer. Show that if $n+1$ distinct integers are chosen from the set $\{1,2,\cdots,2n\}$, then there are always two which differ by 1. [10 marks]

Question 7

Solve the recurrence relation $h_{n+1}=2h_n-1, n\geq 0$, with initial value $h_0=4$. [10 marks]

Question 8

Evaluate the sum

$$\sum_{k=1}^{100}(-1)^k\binom{100}{k}2^k \ . \quad \text{[10 marks]}$$

Question 9

In how many ways can 12 people be divided into 6 pairs? [10 marks]

Question 10

Answer the following questions briefly.

(1) Which complete graphs K_n, $n\geq 2$, have Eulerian cycles? [5 marks]

(2) What is the smallest number of edges that can be removed from K_5 in order to leave a bipartite graph? [5 marks]