國 立 宜 蘭 大 學

１０１學年度研究所碩士班考試入學

工程數學試題

（電子工程學系碩士班）

准考證號碼：

《作答注意事項》

1. 請先檢查准考證號碼、座位號碼及答案卷號碼是否相符。

2. 考試時間：100 分鐘。

3. 本試卷共有 13 題。單選題 8 題，一題 5 分，計算題 5 題，一題 12 分，共計 100 分。

4. 請將答案寫在答案卷上。

5. 考試中禁止使用大哥大或其他通信設備。

6. 考試後，請將試題卷及答案卷一併繳交。

7. 本試卷採雙面影印，請勿漏答。

8. 應試時不得使用電子計算機。

Part 1. 單選題 (每題五分，答錯倒扣一分)

1. The general solution of $ydx + xdy = 0$ is (A) $y = \dfrac{c}{x}$ (B) $\dfrac{x}{y} = c$ (C) $y = ce^x$ (D) $y = c\ln(x)$ (E) $y = ce^{-x}$ (The symbol c is a constant)

2. For the differential equation $y'' - 2y' + y = 4e^x$, it is with the general solution
$y(x) = y_h(x) + y_p(x) = c_1 e^x + c_2 x e^x$. c_1 and c_2 are two arbitrary constants. $y_p(x)$ indicates the particular solution of the differential equation, $y_p(x) = ?$ (A) $2x^2 e^x$ (B) $2e^x$ (C) $2xe^x$ (D) $2e^x + 2xe^x$ (E) 0

3. Which of the following is the Laplace transform of function $\cos(2t)$? (A) $\dfrac{1}{s^2}$ (B) $\dfrac{2}{s^2 + 4}$ (C) $\dfrac{1}{s^2 - 4}$ (D) $\dfrac{s}{s^2 - 4}$ (E) $\dfrac{s}{s^2 + 4}$

4. Which one is the solution of the partial differential equation $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} = 0$? (A) $u(x,y) = Ce^{K(x+y)}$ (B) $u(x,y) = Ce^{K(x-y)}$ (C) $u(x,y) = Ce^{kxy}$ (D) $u(x,y) = Ce^{Kx/y}$ (E) $u(x,y) = Ce^{K(x^2+y^2)}$ (Here C and k are constants)

5. Let $u = (1,0,1)$ and $v = (1,1,0)$. Which of the following vectors is not in Span{u,v}? (A) (1,-1,2) (B) (4,3,1) (C) (1,1,1) (D) (1,2,-1) (E) None of these.

6. For what value of c is the set of vectors {(1, 2, 3, 1), (1, 3, 3, 2), (1, 5, 6, 7), (1, 1, 5, c)} linearly dependent? (A) 0 (B) 2 (C) 5 (D) 7 (E) 9

7. The columns of which matrix form a basis for $\mathbf{R}^3$? (A) $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{bmatrix}$ (C) $\begin{bmatrix} 3 & 4 & 1 & 3 \\ 2 & -6 & 0 & 0 \\ 1 & -5 & 7 & 4 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 3 & 1 & 3 \end{bmatrix}$ (E) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 3 & 1 \end{bmatrix}$

背面尚有試題

8. Let $A = \begin{bmatrix} 5 & 0 & 0 & 0 & 5 \\ 1 & 4 & 6 & 2 & 3 \\ 3 & 4 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 4 \\ 2 & 3 & 0 & 2 & 7 \end{bmatrix}$. The determinant of this matrix is

(A) -600   (B)   600   (C) -120   (D) 120   (E) 0

Part 2. 計算題 (每題十二分)

1. Solve the differential equation $y''+4y = 2x$, $y(0) = y'(0) = 1$

2. (i) Calculate the in Laplace transform $t e^{-2t}$ (ii) Calculate the inverse Laplace transform for $\dfrac{e^{-2s}}{s^3}$

3. A periodic function $f(x)$ with period $T = 2\pi$ is defined as $f(x) = x,\ -\pi < x < \pi$. This function can be represented by Fourier series:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos(n\,x) + b_n \sin(n\,x)\right).\ \text{Find } a_0, a_n \text{ and } b_n$$

4. Let

$$A = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$$

Find a diagonal matrix D and a matrix S such that $A = S^{-1}DS$

5. Let

$$A = \begin{bmatrix} 1 & -1 & 4 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ 1 & -1 & 0 \end{bmatrix}$$

Find an orthonormal basis for the column space of A.