(103)輔仁大學碩士班招生考試試題	考試日期：103 年 03 月 7 日第 2 節
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科目： 軟體與數學　　系所組：資訊工程研究所

1. (10%) Analyze running times of the following two programs.

(a)
```
void programA (int n)
{
    int a = 0;
    for (int i = n; i > 0; i = i / 2 )
      a = a + 1;
}
```

(b)
```
long programB (long n)
{
    if ( n == 1 )
        return 1;
    else
        return (n * programB(n-1));
}
```

2. (40%) Algorithm design and analysis.

(a) (10%) Given a set S and a number m, design an algorithm to determine whether there are two numbers x and y in the set S, such that x+y=m. Ex: Given a set S={1, 2, 3, 4, 5} and m=6, the answer is true because "1+5=6" satisfies the condition "x+y=6" in the set S, where x=1 and y=5. "2+4=6" also satisfies the condition "x+y=6" in the set S. Additionally, analyze the running time of your algorithm.

(b) (10%) Given a positive integers array A[1,…,n]={a_1, a_2, a_3,…, a_n}. Design an algorithm to determine whether any three consecutive integers appear in the array A an ascending order (from small to big). Ex: Given an array A={1, 6, 2, 4, 5, 3}, the answer is true because {1, 2, 3} are three consecutive integers and appear in the array A an ascending order. Additionally, analyze running time of your algorithm.

(c) (10%) Let A[1,…,n] be an array of n integers such that A[1] < A[2] < ・ ・ ・ < A[n]. Design an algorithm to determine whether there exists a "A[i]=i". Ex: Given an array A={1, 3, 4, 5, 9}, the answer is true because A[1]=1. Additionally, analyze the running time of your algorithm.

(d) (10%) It is trivial that there are 2^k different k-bits strings. Let $n=2^k-1$. An array A[1,…,n] contains n different k-bits strings. Design an algorithm to find which k-bits string isn't contained in the array A. The only operation, called **fetch(i, j),** you can apply to access the array A is "fetch the j-bit of A[i]" and this **fetch(i, j)** operation takes constant time. Ex: Given k=3 and 7 different 3-bits strings. Assume A[1,…,7]={"010", "001", "101", "110", "111", "000", "011"}. You are required to find 3-bits string "100" isn't contained in the array A. In this example, **fetch(2,0)** will return "1". Additionally, analyze the running time of your algorithm.

※ 注意：1.考生須在「彌封答案卷」上作答。
2.本試題紙空白部份可當稿紙使用。
3.考生於作答時可否使用計算機、法典、字典或其他資料或工具，以簡章之規定為準。

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3. (6%) Given the linear system: $\begin{cases} 2x + ay = 1 \\ ax + 2y = 1 \end{cases}$

   (a) Determine a particular value of a so that the system has infinitely many solutions.

   (b) Determine a particular value of a so that the system has no solution.

   (c) Determine a particular value of a so that the system has a unique solution.

4. (10%) Find the inverse of the matrix $\mathbf{A} = \begin{bmatrix} 1 & 0 & 5 \\ 1 & 1 & 0 \\ 3 & 2 & 6 \end{bmatrix}$, if it exists.

5. (6%) Determine if the columns of $\mathbf{A} = \begin{bmatrix} 0 & 1 & 4 \\ 1 & 2 & -1 \\ 5 & 8 & 0 \end{bmatrix}$ are linearly independent.

6. (6%) Show that the transformation T defined by $T(x_1, x_2, x_3) = (x_1 + x_2 - 1, x_2 + x_3 - 2, x_3 + x_1 - 3)$ is not linear.

7. (12%) Let $\mathbf{A} = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$.

   (a) (6%) Find a basis for the column space **A**, $C(\mathbf{A})$.

   (b) (6%) Find a basis for the null space **A**, $N(\mathbf{A})$.

8. (10%) Find the eigenvalues of $\mathbf{A} = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$.

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