(103)輔仁大學碩士班招生考試試題	考試日期：103 年 3 月 7 日第 2 節
	本試題共 1 頁

科目：線性代數　　系所組：電機工程(丙組)

1. Find the values of κ such that (a) if $\mathbf{z}$ is in Span$\{\mathbf{x}, \mathbf{y}\}$, and (b) if $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly dependent. (10%)

$$\mathbf{x}=\begin{bmatrix}-1\\2\\4\end{bmatrix},\quad \mathbf{y}=\begin{bmatrix}3\\-6\\-12\end{bmatrix},\quad \mathbf{z}=\begin{bmatrix}-4\\10\\\kappa\end{bmatrix}$$

2. Determine formulas for A, B, C in terms of X, Y, Z. (15%)

$$\begin{bmatrix}I & 0\\0 & I\end{bmatrix}=\begin{bmatrix}A & B\\0 & C\end{bmatrix}\begin{bmatrix}X & Y\\0 & Z\end{bmatrix}$$

3. Find an LU factorization of the given matrix D. Note that L is a lower triangular matrix and U is an upper triangular matrix. (10%)

$$D=\begin{bmatrix}2 & 3 & 2\\6 & 11 & 4\\4 & 5 & -6\end{bmatrix}$$

4. Let

$$E=\begin{bmatrix}2 & 5 & -7 & -3\\1 & 2 & -1 & 9\\2 & 3 & 3 & 5\end{bmatrix}$$

(a) Find the column space and the rank of E. If the column space of E is a subspace of $\mathbb{R}^n$, what is n?

(b) Find the null space and the nullity of E. If the null space of E is a subspace of $\mathbb{R}^m$, what is m? (24%)

5. Find the eigenspace corresponding to each eigenvalue of the matrix M shown below. (16%)

$$M=\begin{bmatrix}-4 & 2\\3 & 1\end{bmatrix}$$

6. Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ and $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2\}$ be bases for $\mathbb{R}^2$. Determine the change-of-coordinates matrix $\mathcal{B}$ to $\mathcal{C}$ and the change-of-coordinates matrix $\mathcal{C}$ to $\mathcal{B}$. (10%)

$$\mathbf{b}_1=\begin{bmatrix}2\\2\end{bmatrix},\quad \mathbf{b}_2=\begin{bmatrix}1\\2\end{bmatrix},\quad \mathbf{c}_1=\begin{bmatrix}-1\\7\end{bmatrix},\quad \mathbf{c}_2=\begin{bmatrix}1\\-8\end{bmatrix}$$

7. Find an orthonormal basis for the column space of the given matrix H. (15%)

$$H=\begin{bmatrix}2 & 5 & 1\\-1 & 0 & -2\\0 & -1 & 3\\1 & 2 & 2\end{bmatrix}$$

※ 注意：1.考生須在「彌封答案卷」上作答。
2.本試題紙空白部份可當稿紙使用。
3.考生於作答時可否使用計算機、法典、字典或其他資料或工具，以簡章之規定為準。