國立臺南大學103學年度 **資訊工程學系碩士班** 招生考試 離散數學與線性代數 試題卷

一、離散數學 (50%)

1. Four machines, *A*, *B*, *C*, and *D*, are connected on a computer network. It is feared that a computer virus may have infected the network. Your security team makes the following statements: (10%)
   (1) If *D* is infected, then so is *C*.
   (2) If *C* is infected, then so is *A*.
   (3) If *D* is infected, then *B* is clean but *C* is infected.
   (4) If *A* is infected, then either *B* is infected or *C* is clean.
   Assume that all statements (1)(2)(3)(4) are true. Which machine are infected or clean you can conclude?

2. Let $\varnothing$ denote the empty set that contains no elements. Prove that for any universe U, $\varnothing$ is a subset of any set $A \subseteq$ U. (10%)

3. (a) Apply Euclidean algorithm to find the greatest common divisor of 630 and 1155. (5%)
   (b) Apply Fundamental Theorem of Arithmetic to find the least common multiple of 242 and 440. (5%)

4. How many ways are there to place 12 marbles to 6 containers if
   (a) Each marble is in different color and the containers are all distinct. (5%)
   (b) The marbles are in the same color and the containers are all distinct. (5%)
   (c) Each marble is in different color, the containers are all distinct and no container is left empty. (5%)
   (d) The marbles are in the same color, the containers are all distinct and no container is left empty. (5%)

二、線性代數 (50%)

1. Determine a basis for each of the following subspaces of R^4. (10%)

   (a) the set of vectors of the form $(a, a - 2b, b, a + b)$

   (b) the set of vectors of the form $(2a, b, a + 2c, a - b)$

2. In each part, use the information in the table to determine whether the linear system $A\mathbf{x} = \mathbf{b}$ is consistent. If so, state the number of solutions it has. (10%)

	(a)	(b)	(c)	(d)	(e)
Size of A	3×3	3×3	3×3	5×6	5×6
Rank(A)	3	2	2	5	4
Rank[A|b]	3	3	2	5	5

3. Evaluate the determinant $\begin{vmatrix} 1 & -1 & 0 & 2 & -1 \\ -3 & 1 & 1 & 0 & 0 \\ 2 & 0 & 4 & 1 & 0 \\ -1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 0 & 3 \end{vmatrix}$ (10%)

4. Consider the following homogeneous system $A\mathbf{x} = \mathbf{0}$, please find a basis for the null space of the matrix A. (10%)

$$\begin{cases} x_1 - x_2 + x_3 - x_4 + x_5 = 0 \\ 4x_2 - 2x_3 - x_5 = 0 \\ x_1 + 3x_2 - x_3 - x_4 = 0 \\ -x_1 - x_2 - x_3 - x_5 = 0 \end{cases}$$

5. Let R^3 have the inner product where $\mathbf{u} = (u_1, u_2, u_3)$,. Use the Gram-Schmidt

   process to transform , $\mathbf{u}_2 = (1, -1, 0)$, $\mathbf{u}_3 = (1, 1, -1)$ into an orthogonal basis. (10%)