

1. Find the limit  $\lim_{x \rightarrow 0^+} (1+x)^{1/x}$ . (10 分)

2. Let

$$f(x) = \begin{cases} x^2, & x \geq 0, \\ -x^2, & x < 0. \end{cases}$$

Compute  $f'(x)$  for all  $x \in \mathbb{R}$ . (10 分)

3. Sketch the region bounded by the curves  $x = y^2$  and  $x - y = 2$ , and find its area. (10 分)

4. Assume that  $u = u(x, y)$  is differentiable. Let  $x = r \cos \theta$  and  $y = r \sin \theta$ . Express

$$\left( \frac{\partial u}{\partial r} \right)^2 + \frac{1}{r^2} \left( \frac{\partial u}{\partial \theta} \right)^2$$

entirely in term of  $\frac{\partial u}{\partial x}$  and  $\frac{\partial u}{\partial y}$ . (10 分)

5. (a) Evaluate  $\int_2^\infty \frac{1}{x(\ln x)^2} dx$ . (10 分)

(b) Determine whether the series  $\sum_{k=2}^\infty \frac{1}{k(\ln k)^2}$  converge. (5 分)

6. Evaluate the double integral  $\iint_{\Omega} xy \, dx dy$ , where  $\Omega$  is the first quadrant region

bounded by curves  $x^2 + y^2 = 4$ ,  $x^2 + y^2 = 9$ ,  $x^2 - y^2 = 1$ ,  $x^2 - y^2 = 4$ . (15 分)

7. Please give a real function which is continuous at 0 but not differentiable at 0. (10 分)

8. Prove that  $e^x > 1 + x$  for all  $x > 0$ . (10 分)

9. Suppose that  $f$  is continuous on  $(-\infty, \infty)$ . Find  $F'(x)$  if

$$F(x) = \int_{x^2}^1 f(y) dy. \quad (10 \text{ 分})$$