國立臺灣海洋大學103學年度研究所碩士班招生考試試題

考試科目：工程數學

系所名稱：光電科學研究所碩士班

1.答案以橫式由左至右書寫。2.請依題號順序作答。

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1. Given that $f(t)=1$, at $t=0$, solve the following differential equations:

   (a) $\dfrac{df(t)}{dt}=-t^2$ (4%)

   (b) $\dfrac{df(t)}{dt}=-f(t)$ (4 %)

   (c) $\dfrac{df(t)}{dt}=[f(t)]^2$ (4 %)

2. Given that $f(t)=1$ and $\dfrac{df(t)}{dt}=0$ at $t=0$, solve the following differential equations:

   (a) $\dfrac{d^2f(t)}{dt^2}=-f(t)$ (4 %)

   (b) $\dfrac{d^2f(t)}{dt^2}=f(t)$ (4 %)

3. Solve the partial differential equation

   $$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]g(x,y)=0$$

   in the range of $0\le x\le 1$ and $0\le y\le 1$, with boundary conditions:

   $g(0,y)=0$, $g(1,y)=0$, $g(x,1)=0$, $g(x,0)=\sin(\pi x)+\sin(2\pi x)$ (7%)

4. Find power series solutions for the Hermite differential equation

   $$\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2\alpha y=0$$

   where α is an arbitrary real number. (6 %)

5. If $z=x+iy$，please show that (a) $\sin(z)=\sin(x)\cosh(y)+i\cos(x)\sinh(y)$;

   (b) $\cos(z)=\cos(x)\cosh(y)-i\sin(x)\sinh(y)$. (8%)

6. Determine the nature of singularities of each of the following functions and evaluate the residues (a>0). (a) $\dfrac{1}{z^2+a^2}$. (b) $\dfrac{z^2}{\left(z^2+a^2\right)^2}$. (c) $\dfrac{ze^{iz}}{z^2+a^2}$. (15%)

7. Prove that $\displaystyle\int_{-\infty}^{+\infty}\frac{\sin^2 x}{x^2}dx=\frac{\pi}{2}$, *Hint.* $\sin^2 x=\frac{1}{2}\left(1-\cos 2x\right)$. (10%)

8. $A=\begin{bmatrix}1 & -1 & -1\\ 1 & 3 & 1\\ -3 & 1 & -1\end{bmatrix}$ 求矩陣P使得$P^{-1}AP$為對角矩陣 (10%)

9. $A=\begin{bmatrix}1 & -2 & 2\\ 0 & 3 & 2\\ 1 & 0 & 1\end{bmatrix}$, $B=\begin{bmatrix}2 & 0 & 1\\ 0 & -1 & -2\\ 3 & 1 & -2\end{bmatrix}$求 |A|、|B| 與 |AB| (12%)

10. $\langle p\,,q\rangle = a_0b_0 + a_1b_1 + \cdots + a_nb_n$為一內積函數

令$p(x)=1-2x^2$, $q(x)=4-2x+x^2$ 為$P_2(x)$上的向量

(a) $\langle p\,,q\,\rangle = ?$ (b) $\|q\| = ?$ (c) $d(p\,,q)=?$ (12%)