

國立臺灣海洋大學 103 學年度研究所碩士班招生考試試題

考試科目：工程數學(含常微分方程、線性代數及拉式轉換)

系所名稱：通訊與導航工程學系碩士班電子導航與定位組

1. 答案以橫式由左至右書寫。2. 請依題號順序作答。

1. Solve the following ordinary differential equations:

(a) (10%) $(4xy + 3y^2 - x)dx + x(x + 2y)dy = 0$

(b) (10%) $y'' - 2y' + y = e^x + x$

2. (10%) Solve the initial value problem

$$y'' - y' - 2y = 3e^{2t}, \quad y(0) = 1, \quad y'(0) = -2.$$

3. (10%) Solve the initial value problem

$$\begin{cases} \frac{dy_1}{dt} = y_1 - 4y_2 \\ \frac{dy_2}{dt} = 4y_1 - 7y_2 \end{cases}, \quad y_1(0) = 3, \quad y_2(0) = 2.$$

4. (10%) Let $H(s) = \frac{1}{s^2(s-a)}$. Find $h(t)$ using the Convolution theorem of the

Laplace transform.

5. (10%) Solve the integro-differential equation by Laplace transform

$$y'(x) = 1 - \int_0^x y(t)e^{-2(x-t)} dt, \quad y(0) = 1.$$

6. (10%) Solve the following simultaneous equations by Gauss elimination.

$$3x_1 - 2x_2 + 2x_3 = 10$$

$$x_1 + 2x_2 - 3x_3 = -1$$

$$4x_1 + x_2 + 2x_3 = 3$$

7. Let $\mathbf{A} = \begin{bmatrix} 5 & 6 \\ -2 & -2 \end{bmatrix}$

- (a) (10%) Find the eigenvalues and the corresponding eigenvectors of \mathbf{A} .
- (b) (5%) Find an invertible matrix \mathbf{S} such that $\mathbf{S}^{-1}\mathbf{A}\mathbf{S}$ is diagonal.
- (c) (5%) Use result from (b) to compute \mathbf{A}^{100} .

8. Prove (a) (5%) $(\mathbf{A}^T)^{-1} = (\mathbf{A}^{-1})^T$.

(b) (5%) $(\mathbf{A}\mathbf{B})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$.

where \mathbf{A} and \mathbf{B} are nonsingular matrices.