

考試科目：線性代數

系所名稱：電機工程學系碩士班控制組

1. 答案以橫式由左至右書寫。2. 請依題號順序作答。

-
- (1) (20 %) Let V, W be vector spaces over the same field and $L : V \rightarrow W$ a linear mapping from V to W . Prove that the kernel $\ker L$ is a subspace of V and the image $\text{im}L$ is a subspace of W .
- (2) (20 %) Consider the differentiator mapping $D : \mathbb{R}_3[t] \rightarrow \mathbb{R}_3[t]$ defined by $D(f) = f'$, where f' denotes the derivative of f , and $\mathbb{R}_3[t]$ the space of all polynomials with real coefficients of degree at most 2.
- (a) Find an ordered basis \mathcal{B} such that the representation matrix $[D]_{\mathcal{B}}$ is a Jordan matrix.
- (b) Find the minimal polynomial of D .
- (3) (20 %) Consider the ordered basis $\mathcal{B} = \{(1, 0, 1), (1, 2, 1), (0, 0, 1)\}$ of \mathbb{R}^3 . Find the dual basis of \mathcal{B} .
- (4) (20 %) Given $\mathbf{A} = \begin{bmatrix} 5 & 4 \\ 4 & -1 \end{bmatrix}$, find an orthogonal matrix P for which $P^T \mathbf{A} P$ is diagonal.
- (5) (20 %) Given $\mathbf{A} = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \end{bmatrix}$, find the singular value decomposition of \mathbf{A} .