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國立聯合大學 101 學年度碩士班考試招生

電機工程研究所 入學考試試題

科 目：工程數學 第 1 頁共 2 頁

『***Show the details of your work.***』

1. Find the Laplace transform, if：

   (A). **(5%)** $f(t)=\delta(t-a)$.

   (B). **(10%)** $f(t)$ is a piecewise continuous function with period T.

2. Consider a linear ODE $y''(t)+6y'(t)+9y(t)=r(t)$.

   (A). **(5%)** If $r(t)=0$, find a general solution.

   (B). **(10%)** Suppose y_1, y_2 are linearly independent solutions and $r(t)=2e^{-3t}$. Find a particular solution in the form $y_p=u_1y_1+u_2y_2$.

3. Given $f(z)=\dfrac{1}{z^2(z-1)}$,

   (A). **(5%)** Find the residues at its poles.

   (B). **(10%)** Determine the value of $\int_C f(z)dz$, when C is the circle $\left|z-\dfrac{1}{3}\right|=1$.

4. Determine the Fourier transform and roughly sketch the results.

   (A). **(5%)** The function $x(t)=e^{-2\cdot|t|}$.

   (B). **(10%)** The periodic function $y(t)=5\cdot\cos(\omega\cdot t)$ with $\omega=2$ rad/s.

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5. **(20%)** Consider the following linear system $Ax = b$.

$$\begin{bmatrix} 1 & 1 & -1 & 2 & 6 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -2 \\ 6 \\ 8 \\ 0 \\ 0 \end{bmatrix}$$

(A). Find the column space C(A).

(B). Find the Null space N(A).

(C). Find the whole solution of the system.

(D). Find the basis of row space C(A^T).

(E). Find the basis of left null space N(A^T).

6. **(20%)** Please indicate whether each of the following statements is always true or sometimes false. Justify your answer by giving a logical argument otherwise the score will not be counted.

(A). In the case of $Ax = b$ is inconsistent, the solution of $A^T A\hat{x} = A^T b$ is better than $A\hat{x} = p$ with $p = A(A^T A)^{-1} A^T b$.

(B). All the vectors in the null space of $A - \lambda_1 I$ are the eigenvectors of eigenvalue λ_1.

(C). If the coefficient matrix of the homogeneous system of equations $Ax = b$ is a square matrix with determinant $|A| \neq 0$, hen a solution is uniquely given.

(D). For a linear non-homogeneous system $Ax = b$, if we have $rref\,(A) = \begin{bmatrix} I & F \\ 0 & 0 \end{bmatrix}$, that means the system always have infinitely many solutions.

(E). If the dimension of null space $N(A - \lambda I)$ is more than one, imply that A has repeated eigenvalues.