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國立聯合大學 101 學年度碩士班考試招生

光電工程學系碩士班　入學考試試題

科　目：工程數學　　第 1 頁共 1 頁

1. Solve the **Bernoulli equation** (10%)

$y' + xy = xy^{-1}, \quad y(0) = 3.$

2. Solve the initial value problem (10%)

$(e^x + 3y^2)dx + 2xydy = 0, \qquad y(0) = 2.$

3. Solve the initial value problem by the Laplace transform. (20%)

$y''' - y' = sin\,(t)\ , \qquad y(0) = 2,\ y'(0) = 0,\ y''(0) = 1.$

4. Let $f = 4x^2 + xy^2 + 9y^3z^2$ and $\vec{k} = xz\,\hat{x} + (x-y)^2\,\hat{y} + 2x^2yz\,\hat{z}$. (20%)

Find

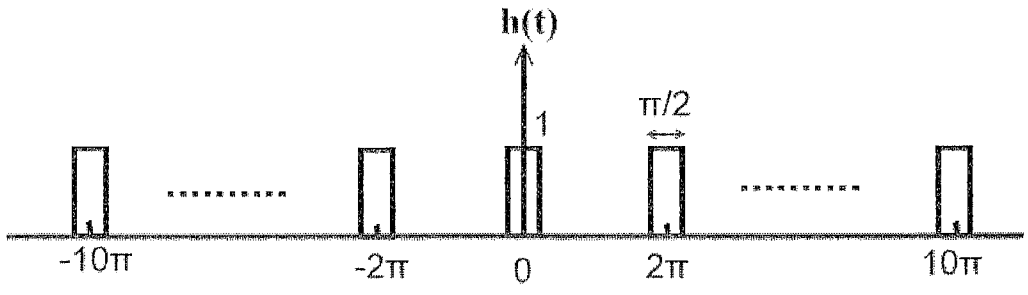
(a) $\nabla^2 f$

(b) $curl(grad\ f)$

(c) $\nabla f \cdot curl\,\vec{k}$

(d) The unit vector of f pointing in the direction of steepest increase at $(3,1,0)$

5. Find the spectrum of $h(t)$ by Fourier analysis. (20%)



6. $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

(a) Find a orthogonal matrix P and it's inverse matrix P^{-1} to orthogonally diagonalize the given matrix A.

(b) If A is similar to a diagonal matrix D, find D.

(c) Find A^{50}. (Just write down the expressions. You need not to compute the result.)

(20%)