

所別：財務金融學系碩士班 乙組(一般生) 科目：微積分 共 乙 頁 第 1 頁

本科考試禁用計算器

*請在試卷答案卷(卡)內作答

1. Given an equation

$$P_t = Xe^{-rt}N(-d_2) - S_t N(-d_1),$$

where

$$d_1 = \frac{\ln\left(\frac{S_t}{X}\right) + \left(r + \frac{\sigma_s^2}{2}\right)\tau}{\sigma_s \sqrt{\tau}}, \quad d_2 = \frac{\ln\left(\frac{S_t}{X}\right) + \left(r - \frac{\sigma_s^2}{2}\right)\tau}{\sigma_s \sqrt{\tau}} = d_1 - \sigma_s \sqrt{\tau},$$

$$\tau = T - t, \text{ and } N(d_1) = \int_{-\infty}^{d_1} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du.$$

a. (15%) It can be easily derived that $\frac{\partial N(d_1)}{\partial d_1} = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}}.$

Please show $\frac{\partial N(d_2)}{\partial d_2} = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_2^2}{2}} \cdot \frac{S_t}{X} \cdot e^{rt}.$

b. (15%) Please calculate $\frac{\partial P_t}{\partial S_t}.$ c. (20%) Please calculate $-\frac{\partial P_t}{\partial \tau}.$

2. Consider the problem

$$\text{minimize}_{w_1, w_2, w_3} \sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + w_3^2 \sigma_3^2 + 2w_1 w_2 \sigma_{12} + 2w_1 w_3 \sigma_{13} + 2w_2 w_3 \sigma_{23}$$

subject to

$$w_1 + w_2 + w_3 = 1$$

$$w_1 R_1 + w_2 R_1 + w_3 R_1 = E^*$$

a. (10%) Please write the Lagrangian.

b. (10%) Please show the first order conditions.

參考用

注意：背面有試題

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3. (10%) If $w_1 + w_2 = 1$, please obtain the w_1 and w_2 to minimize the following equation.

$$\sigma_A^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \sigma_{12}$$

4. (10%) $U(W) = \begin{cases} \frac{W^{1-r} - 1}{1-r}, & \text{if } r \neq 1 \\ \ln(W), & \text{if } r = 1 \end{cases}$

Please calculate $R(W) = -\frac{U''(W)}{U'(W)}$.

5. (10%) $F(W) = \frac{1-\gamma}{\gamma} \left(\frac{\alpha W}{1-\gamma} + \beta \right)'$

If $r \neq 1$, $\alpha > 0$, $\left(\frac{\alpha W}{1-\gamma} + \beta \right) > 0$, and $\beta = 1$ if $\gamma \rightarrow -\infty$, please calculate $-W \frac{F''(W)}{F'(W)}$.

參考用

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