國立中央大學103學年度碩士班考試入學試題卷

所別：財務金融學系碩士班 甲組(一般生)　科目：統計　共 2 頁 第 1 頁
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本科考試禁用計算器　**Be sure to provide proof and explanation in your answer.**　＊請在試卷答案卷（卡）內作答

1. Consider a random variable X with p.d.f. $f(x)=\frac{1}{4}e^{-(x-1.5)/4}$, $1.5\le x<\infty$. Find the mean $\mu=E(X)$? (5%)

2. A continuous random variable X has p.d.f. given by

$$f(x)=\begin{cases}k(1-x)x^2 & \text{, if } 0<x<1\\ 0 & \text{, o.w.}\end{cases}$$

(a) Find the constant k　(5%)

(b) Find $f(x\mid x>0.5)$　(5%)

(c) Find $E(X\mid X>0.5)$　(5%)

3. If the joint density function of two random variables x and y is given by

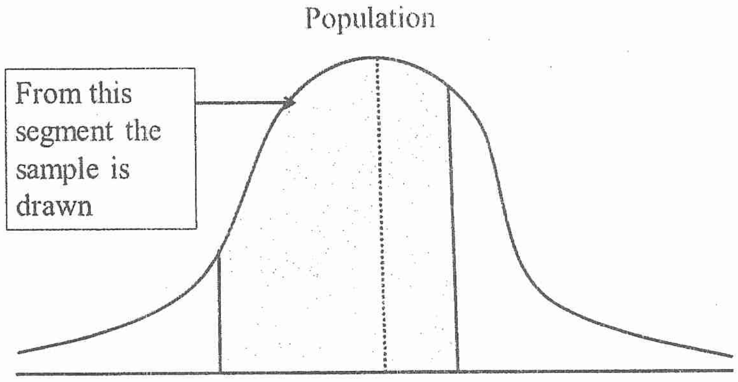
$$f(x,y)=\begin{cases}2(x+2y)/5 & \text{, for } 0<x<1, 0<y<1\\ 0 & \text{, o.w.}\end{cases}$$

Find the conditional mean and the conditional variance of x given y=0.5 (5%)

4. 設二維隨機變數(X,Y)為在 0<x<1, 0<y<1 內之均勻分配，試問：
(a) X 的邊際分配(marginal distribution)為何? (5%)
(b) 令 W=max(X,Y)，則 W 之機率密度函數(p.d.f.)為何？ (5%)
(c) 令 Z=min(X,Y)，則 Z 之機率密度函數為何？ (5%)

5. 設任一試驗均有三種可能結果之一出現，分別為 A, B, C，其出現之機率為 p^2, $p(1-p)$, $1-p$，今若獨立觀察 n 次此種試驗，則
(a) 求此試驗結果出現之機率分配 (5%)
(b) 試求參數 p 之最大概似估計式 (5%)

6. Suppose we are to conduct statistical inference with a given sample. However, the observations in this sample are actually not randomly distributed across the population. All the observations in this sample are drawn from a particular segment of the population, which is illustrated as the shaded area in the following graph:



In this case, when we use the t-statistics calculated from the observations in this sample to test the null hypothesis that population mean equals zero, is it type I or type II error that we are likely to commit, and why? (6%)

參考用

注意：背面有試題

7. Suppose the p.d.f. of a random variable x is $f(x)=1/c$, where $0<x<c$. We are to test the null hypothesis H_0: $c=2$ versus the alternative hypothesis H_1: $c=3$. If we draw an observation of x and reject H_0 if the drawn $x > 3/5$, then:
   (a) What is the probability of committing type I error? (6%)
   (b) What is the probability of committing type II error? (6%)

8. A sample of 25 daily returns for a stock has sample mean $\bar{x}=0.03$. In the two following situations perform the test examining if the population mean of returns is zero at α% level of significance.
   (a) The population variance is 0.009. (Be sure to specify the distribution which your statistics follows.) (5%)
   (b) The population variance is not known and the sample variance is 0.0105. (Be sure to specify the distribution which your statistics follows.) (5%)

9. Suppose we stand at time t=0 and consider the following model for the time-series dynamics of Y_t, t=1,2:

$$Y_1=\beta Y_0+u_1,$$
$$Y_2=\beta Y_1+u_2,$$

where the subscript represents time. Residual terms u_t satisfy

$$E(u_t)=0 \text{ for } t=1,2,$$
$$E(u_t^2)=\sigma^2 \text{ for } t=1,2,$$
$$E(u_1u_2)=\sigma_{12}\neq 0.$$

Y_0 is a known number at time t=0. Find $Cov(Y_1,Y_2)$. (5%)

10. Suppose $Y_i=\alpha+\beta X_i+\varepsilon_i$. Determine whether the least-squares estimate of β is unbiased in the two following situations:
   (a) ε_i is unconditional on X_i and $E(\varepsilon_i)=\gamma$. (6%)
   (b) ε_i is conditional on X_i and $E(\varepsilon_i)=\gamma X_i$. (6%)

11. Suppose a sequence of weekly returns $\{r_t\}$ is distributed with mean 0 and variance σ^2 and there exists correlations between $\{r_t\}$. Define two-week returns as $r(2)_t=r_t+r_{t-1}$ and a variance ratio $VR=\dfrac{Var(r(2)_t)}{2Var(r_t)}$. Show $VR=1+\rho$, where ρ is the correlation between r_t and r_{t-1}. (5%)

注意：背面有試題