【不可使用計算機】*作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

1. Evaluate the limits of the following functions:

(a) (3%) $\lim_{x\to -3}\frac{x^2+x-2}{x^2+5x+6}$

(b) (3%) $\lim_{x\to c}\frac{\frac{1}{x}-\frac{1}{c}}{x-c}$

(c) (4%) $\lim_{x\to 9}\frac{\sqrt{x}-2}{x^2-16}$

2. Evaluate:

(a) (3%) $\int \sin^3 x \cos^8 x\, dx$

(b) (3%) Find the sum of the series $\sum_{n=1}^{\infty}\frac{1+2^{n+1}}{3^n}$

(c) (9%) Solve the integral equation precisely

$$y(x) = 1 + \int_0^x \frac{(y(t))^2}{1+t^2}dt$$

3. Show that

(a) (5%) $\lim_{x\to 0}\frac{x-\sin x}{x^3} = \frac{1}{6}$

(b) (5%) The derivative of $\frac{e^x-e^{-x}}{e^x+e^{-x}}$ is $\frac{4}{(e^x+e^{-x})^2}$

4. Let a_n be defined recursively by

$a_1 = 1,\ a_{n+1} = \sqrt{6+a_n},\ n \in N$

Bounded Monotonic Converge Theorem (BMCT): if a sequence is bounded and increasing, then it converges. Show that

(a) (5%) Sequence $\{a_n\}$ is increasing.

(b) (5%) Sequence $\{a_n\}$ is bounded. (Hint: You can use mathematical induction to prove it)

(c) (5%) $\lim_{n\to\infty} a_n$ exists, then try to find the limit of the sequence $\{a_n\}$.

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5. (20%) The change of the <u>*internal energy U*</u> could be represented as following:

$$dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV$$

where $\left(\frac{\partial U}{\partial V}\right)_T$ indicates how internal energy change with volume under constant temperature and it obeys the equation shown below:

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$$

where P is pressure, T is temperature and V is volume.

(a) Assume argon obeys van der Waals gas equation, show that $\left(\frac{\partial U}{\partial V}\right)_T = \frac{an^2}{V^2}$. (van der Waals gas equation: $\left[P + a\left(\frac{n}{V}\right)^2\right](V - nb) = nRT$, where R is gas constnat and it is $8.3145\ \mathrm{J \cdot K^{-1} \cdot mol^{-1}}$)

(b) Calculate ΔU for the *isothermal* reversible expansion of 1 mole argon (a= 1.337 $\mathrm{atm{\cdot}L^2{\cdot}mol^{-2}}$; b=$3.20\times10^{-2}\ \mathrm{L{\cdot}mol^{-1}}$) for an initial volume of 1.00 L to 24.0 L at *constant temperature* of 298K. (unit conversion: 1 atm·L = 101.325 Joule).

(c) We now explore the effect of the temperature dependence of the heat capacity on the internal energy. $\left(\frac{\partial U}{\partial T}\right)_V$ indicates how internal energy changes with temperature under constant volume and it is the heat capacity at constant volume, $C_V = \left(\frac{\partial U}{\partial T}\right)_V$. Suppose that the molar internal energy of one mole substance over a limited temperature range can be expressed as a polynomial in *T* as $U(T) = a + bT + cT^2$. Find an expression for the constant-volume molar heat capacity $C_{V,m}$ at a temperature *T*.

6. (12%) For DNA helix formation, the following reaction scheme, is usually used:

$$\mathrm{A+B} \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} \text{Unstable Helix} \xrightarrow{k_2} \text{Stable Double Helix}$$

(a) All the steps in the reaction scheme are elementary reactions. Express $\frac{d[A]}{dt}$, $\frac{d[Unstable\ Helix]}{dt}$, and $\frac{d[Stable\ double\ helix]}{dt}$ in terms of reaction constants (k_1, k_{-1} or k_2) and the concentration of all related species ([A], [B] or [$Unstable\ Helix$]).

(b) Using steady-state approximation ($\frac{d[Unstable\ Helix]}{dt}=0$), derive the rate equation for the formation of the double helix and express the rate constant of the reaction in terms of the rate constants of the individual steps.

(c) When [B]>>[A] and [B] is almost unchanged during the reaction. What would be the reaction order for substance A in this reaction?

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7. (18%) For a particle of mass m that is constrained to move freely along a line between 0 and a obeys the following equation:

$$\frac{d^2\psi(x)}{dx^2}+\left(\frac{8\pi^2 mE}{h^2}\right)\psi(x)=0$$

with the boundary condition $\psi(0)=\psi(a)=0$.

The general solution for the equation given above is

$$\psi(x)=A\cos kx+B\sin kx \quad \text{with } k=\sqrt{\frac{8\pi^2 mE}{h^2}}$$

(a) Using the given boundary condition to show that $A=0$ and $E=\frac{h^2n^2}{8ma^2}$, where n is integer.

(b) Using result from (a), show that $\psi(x)=B\sin kx=B\sin\frac{n\pi x}{a}$.

(c) Because that the particle is restricted to the region $(0,\ a)$, it is certain to be found there and so the probability that the particle lies between 0 and a is unity. Here is the normalization equation:

$$\int_0^a \psi^*(x)\,\psi(x)dx=1$$

Using the normalization equation, show that $B=\sqrt{\frac{2}{a}}$.