1. Find a general solution of each of the following equation:

 (1) $y'' + 4y = x + 2e^{-2x}$ (12%)

 (2) $x^2 y'' - 5xy' + 9y = 0$ (12%)

2. Use the Laplace transform to solve the initial value problem: $y'' + y = t; y(0) = 1, y'(0) = 0$ (16%)

3. If matrix $A = \begin{bmatrix} 2 & 0 & 1 & 0 \\ 1 & 3 & 0 & 2 \\ 0 & 1 & -2 & 1 \\ 4 & 0 & 1 & 1 \end{bmatrix}$, find $|A|$ (10%)

4. Let the vector function $\mathbf{v} = [xy, yz, zx]$. Find the divergence and curl of $\mathbf{v}$ at (1,-1,1). (10%)

5. Let $f(x) = x^2$ for $-\pi \le x \le \pi$. Find the Fourier coefficients and Fourier series of f on $[-\pi, \pi]$. (20%)

6. Solve $\dfrac{\partial u}{\partial t} = c^2 \dfrac{\partial^2 u}{\partial x^2}$ $(0 < x < L, t > 0)$

 $u_x(0,t) = u_x(L,t) = 0$ $(t > 0)$

 $u(x,0) = f(x)$ $(0 < x < L)$ (20%)