

【可使用計算機】\*作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符!!

1. Evaluate the limit, if it exists.

(a) (5 points)  $\lim_{x \rightarrow 1} \frac{|x-3|-2|2x-1|}{x-1}$ .

(b) (5 points)  $\lim_{x \rightarrow 0} \frac{1-\cos(\sin x)}{x^2}$ .

2. Evaluate the integral.

(a) (5 points)  $\int_1^3 \frac{x^2+3x+1}{\sqrt{x(x+1)^2}} dx$ .

(b) (5 points)  $\int_0^1 (\sin^{-1} x)^2 dx$ .

3. (10 points) Find values of  $a$  and  $b$  for which  $\lim_{x \rightarrow 0} \left( \frac{\sin 2x}{x^3} + \frac{a}{x^2} + \frac{b}{x} \right)$  is finite and evaluate the limit.

4. (10 points) How many real roots does  $x^5 - 80x + 1 = 0$  have? Explain your answer.

5. (10 points) Find three positive numbers  $x$ ,  $y$  and  $z$  whose sum is 12 such that the product  $xy^2z^3$  is a maximum.

6. (10 points) Let  $a_0 = 2$ , and let  $a_n$  be defined recursively by

$$a_n = \frac{2 + 3a_{n-1}}{4 + a_{n-1}}, \text{ for } n = 1, 2, 3, \dots$$

Decide whether the limit  $\lim_{n \rightarrow \infty} a_n$  exists. If it exists, find its value.

7. (10 points) Evaluate the integral  $\iiint_R |xyz| dv$ , where  $R$  is the solid ellipsoid  $\frac{x^2}{1^2} + \frac{y^2}{2^2} + \frac{z^2}{3^2} \leq 1$ .

8. (10 points) Find the volume of the solid bounded by the cone  $z^2 = x^2 + y^2$  and the paraboloid  $4z = x^2 + y^2$ .

9. (10 points) Find the equation of the tangent plane of the surface  $\sin(xyz) = -2x + y - z$  at the point  $(1, 2, 0)$ .

10. (10 points) Find the maximum and minimum values of  $f(x, y) = (x^2 + y^2)e^{y^2 - x^2}$  on the disk  $D = \{(x, y) : x^2 + y^2 \leq 4\}$ .