

國立臺北大學 103 學年度碩士班一般入學考試試題

系(所)組別：通訊工程學系

科 目：機率

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可 不可使用計算機

1. If the moment-generating function of a random variable X is

$$M(t) = \exp(5t + 12t^2),$$

find the variance $\text{Var}(X)$. (10%)

2. Random variables X and Y have the joint probability density function

$$f_{X,Y}(x,y) = \begin{cases} 1/2 & -1 \leq x \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the conditional expectation $E[X | Y = y]$. (10%)

3. X_1 follows an exponential distribution with parameter $\lambda_1 = 2$, and independently, X_2 follows an exponential distribution with parameter $\lambda_2 = 3$. Find the probability $P(X_1 < X_2)$. (10%)

4. Let X, Y have the joint probability density function

$$f_{X,Y}(x,y) = 2, \quad 0 < x < y < 1.$$

Consider the transformation

$$W = \frac{X}{Y}, \quad Z = Y.$$

Find the joint probability density function of W, Z . (10%)

5. X and Y are zero mean independent Gaussian random variables with common variance σ^2 . Find the probability density function $f_Z(z)$ for $Z = X^2 + Y^2$. (10%)

6. There are 65 Japan cars, 20 German cars and 15 Taiwan cars in a car rental company. Suppose we want to rent 10 different cars from the company. Let Y be the number of German cars selected. Find the probability $P[Y=k]$. (The answer can be in an equation form. Do not need to calculate the value). (10%)

7. A non-symmetric binary communication channel is shown in Fig. 1: If the input is "0", there is a probability " $1 - \epsilon_1$ " that the output is "0" and a probability " ϵ_1 " that the output is "1". If the input is "1", there is a probability " ϵ_2 " that the output is "0" and a probability " $1 - \epsilon_2$ " that the output is "1". Assume that the input is "0" with probability " p " and "1" with probability " $1-p$ ". Find the probability that the output is "1". (10%)

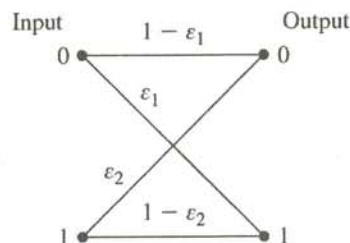


Fig. 1

8. Let X be the number of heads in three independent tosses of a fair coin. Find the probability mass function (pmf) of X . (10%)

9. Two fair dice are tossed. Let X be the total number of dots on the up-side faces. What is the expected value of X ? (10%) and what is the variance of X ? (10%)

試題隨卷繳交