國立臺北大學103學年度碩士班一般入學考試試題

系（所）組別：資訊工程學系

科　　　目：線性代數與離散數學

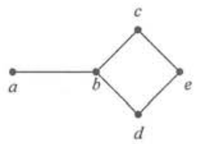
□可 ☑不可使用計算機

1. (15%) Use the pigeonhole principle to show that in a sequence of 10 distinct integers, there is either an increasing subsequence of length 4 or a decreasing subsequence of length 4.

2. (15%) Let $\boldsymbol{R}$ be the relation on the set of integers such that $a\boldsymbol{R}b$ if and only if 6 divides $(a-b)$.
   (a) Show that the relation $\boldsymbol{R}$ is an equivalence relation.
   (b) What are the equivalence classes?

3. (10%) There are two boxes, one red and one blue, and in the red box we have 2 apples and 6 oranges, and in the blue box we have 3 apples and 1 orange. Let us suppose that we pick the red box 40% of the time and we pick the blue box 60% of the time, and that when we remove an item of fruit from a box we are equally likely to select any of the pieces of fruit in the box. Given that we have chosen an orange, what is the probability that the box we chose was the blue one?

4. (10%) Draw all the spanning trees of the following graph.



5. (5%) Suppose $A=\begin{bmatrix}-1 & 0 & 1\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$. The dimension of the eigenspace corresponding to the eigenvalues $\lambda=1$ is __________.

6. (5%) Suppose $A=\begin{bmatrix}-2 & -6\\ 1 & 3\end{bmatrix}$. Then, $(e^{A})^{-1}$ is __________.

7. (10%) Compute the following sum of determinants.

$$\begin{vmatrix}2 & 0 & 0 & 1\\ 0 & 1 & 0 & 0\\ 1 & 6 & 2 & 0\\ 1 & 1 & -2 & 3\end{vmatrix}+\begin{vmatrix}2 & 1 & 2 & 1\\ 3 & 0 & 1 & 1\\ -1 & 2 & -2 & 1\\ -3 & 2 & 3 & 1\end{vmatrix}$$

8. (10%) Given $A=\begin{bmatrix}1 & 4 & 3\\ -1 & -2 & 0\\ 2 & 2 & 3\end{bmatrix}$. There are elementary matrices E_1, E_2, E_3 such that $E_3E_2E_1A=U$, where U is an upper triangular matrix. Compute $L=E_1^{-1}E_2^{-1}E_3^{-1}$.

9. (10%) Let L be the linear transformation on R^3 defined by $L(\vec{x})=(2x_1-x_2-x_3,\ 2x_2-x_1-x_3, 2x_3-x_1-x_2)^T$ and let A be the standard matrix representation of L. If $\vec{u_1}=[1,1,0]^T$, $\vec{u_2}=[1,0,1]^T$, $\vec{u_3}=[0,1,1]^T$, then $[\vec{u_1},\vec{u_2},\vec{u_3}]$ is an ordered basis for R^3 and $U=(\vec{u_1},\vec{u_2},\vec{u_3})$ is the transition matrix corresponding to a change of basis from $[\vec{u_1},\vec{u_2},\vec{u_3}]$ to the standard basis $[\vec{e_1},\vec{e_2},\vec{e_3}]$. Determine the matrix B representing L with respect to the basis $[\vec{u_1},\vec{u_2},\vec{u_3}]$ by calculating $U^{-1}AU$.

10. (10%) Find the equation of the circle that gives the best squares circle fit to the n sample pairs of coordinates $(x_1,y_1),(x_2,y_2),\cdots,(x_n,y_n)$.

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